MATHEMATICAL EDUCATION IN THE AMERICAS II

A Report of the Second Inter-American Conference on Mathematical Education

Lima, Peru, December 4 – 12, 1966

Edited by

HOWARD F. FEHR

Program of Mathematics

Teachers College, Columbia University

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FOREWORD

The first Inter-American Conference on Mathematical Education was held at Bogota, Colombia, from December 4 to 9, 1961. At this conference, the first of its kind in convening mathematicians from twenty-three countries of the Western Hemisphere, the primary purpose was to bring to the attention of all the participants the changes in mathematical knowledge that entail a need for reform of school and university curriculums, and thus to motivate all countries to initiate reform activities. The deliberations of this conference were published in the report Mathematical Education in the Americas.[1]

After a period of four years of intense efforts in some countries and sporadic activity in the other countries, the Inter-American Committee on Mathematical Education, established in Bogota in 1961, felt the time had come to make an evaluation of these activities. There was need to establish consolidation of efforts, means of inter-country communication and to plan, continent wise, for the mathematical education that must be provided for economic and social growth of each nation. In the late Fall of 1965, the Committee began seeking funds and other means for a second Inter-American Conference on Mathematical Education. This conference was to address itself to three special themes: A review and an examination of the current

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[1] Mathematical Education in the Americas, Edited by Howard F. Fehr, Bureau of Publications, Teachers College, Columbia University, New York, 1962.

problems involved in developing mathematical education, especially in Latin America; an examination of the desired curriculum for secondary and undergraduate university study of mathematics; and the concomitant problem of educating in sufficient quantity and quality the teachers and professors of mathematics for schools and universities.

To achieve this purpose, the Organizing Committee selec outstanding mathematicians and mathematics educators from Europe and the American countries to present papers on the themes. Following the presentations there were informal discussions, groups of the participants were organized to study and report on the three particular aspects of developin mathematical education. From each country there was solicited and distributed to the participants a paper which summarized the activities in the country during the last five years and the status of mathematical education as of the end of the year 1966.

The addresses presented in the conference are published in Part II of this report. In Part I there is presented the opening address and charge to the participants. Part III contains a set of abbreviated country reports on the status of mathematics. Part IV gives the conclusions of the conference in a form of a set of resolutions, and the basis on which the new Inter-American Committee on Mathematical Education will operate in the years ahead.

Both the Conference and the publication of English, Spanish, and Portuguese editions of this report were made possible by grants from a number of philanthropic and scientific organizations. The debt of the Organizing Committee and the participants in the Conference for this generous support is duly acknowledged in Part IV. An appendix lists the program, the committees, the participants, and invited speakers and observers. The preparation of the report was under the direction of Howard F. Fehr (U.S.A.), Executive Secretary of the Conference.

The editor acknowledges a great debt to Professor Mordecai Rubin of Teachers College, Columbia University, for his Spanish-English and English-Spanish translations. To his wife Gisela, he expresses his deep appreciation of the hundreds of hours of critical reading and typing of the manuscript.

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PART I

OPENING ADDRESSES

REFORM IN MATHEMATICAL PEDAGOGY

Carlos Cueto Fernandini
Minister of Education
Lima, Peru

The pedagogical work of the second half of the twentieth century is still reeling from that combination of events which we refer to as the Revolution in the Teaching of Mathematics. This revolution first arose in the minds of professional mathematicians who, about 25 years ago, verified -once again-- that the schools of all countries were still dealing with the most obsolete notions in the mathematical sciences. The nearest thing to "new" in school mathematics programs was two hundred years old. Even today, in spite of everything, we still fail to take advantage of the new and marvelous contributions made by mathematical science to the perfecting of the human spirit as well as our material environment. If one of the essential aspects of education is the integration of man and the system of knowledge contemporary to him, how can we turn our back on modern mathematics? How can we move back our horizons to the time that knew nothing, for example, about set theory? To take no notice of recent discoveries in mathematics is like teaching the history of humanity and stopping at the French Revolution.

It is clear that the mathematicians are right. We do have to introduce new ideas into the teaching programs of our present schools or at least give new meaning to the teaching of traditional mathematics. Otherwise, our present-day civilization may well turn out to appear as unintelligible as an eighteenth century city presented in a retrospective novel as equipped with electronic devices to regulate automobile traffic.

The spectacular appearance of Sputnik was not unrelated to this revolution. As has often happened in the history of education, outside events have forced educators to review established practice and uproot their most entrenched prejudices. This has invariably happened in times of intense cultural crisis or rapid social and economic transition. Since it is a symptom of a process that has been repeated many times before, the revolution in the teaching of mathematics is one of the major discoveries in the history of western educational thought.

What makes up such a revolution that in nearly twenty-five years seems to have opened countless possibilities? One of its basic elements is, of course, the modern mathematical sciences, which, in their varied branches, represent (today more than ever) the ability of the human mind to create a language of symbols that is not tied to the particular and the contingent but is capable of rising to necessary rational, and universal, relationships. The same symbolic language is capable of generating endlessly new technical forms.

From the point of view of education, however, this is not the fundamental point. For educators it is more important to know that the new mathematics is oriented toward stimulating within the child thinking and inventive capacities for logical investigation. Under this new pedagogy, mathematics serves to promote the human individual, making the student reason with himself, draw on an inner discipline to confront the most rigorous and demanding symbols, and maintain his freedom before logical structures.

Education is vital to any science. The greatness of education lies in the fact that it takes a general view of knowledge, culture, society, and history, not as unrelated realities, but rather as elements of a single complex, inseparable from the human being - his daily problems, his final destiny and his essential liberty. Actually, the new mathematics (or the traditional, if taught with present-day aims) seeks to restore the Platonic pedagogical ideal of the slave, who is guided by the teacher to discover for himself a mathematical theorem and to formulate it with discipline and precision. It could be maintained, then, that today's revolution revives very old bits of Platonic doctrine. This is true, in part, but there is more. And this extra that lies at the bottom of our current revolution is the fact that the new pedagogy for mathematics has no less a goal than the preparation of today's child for a life of dignity in a technologically based world which constantly threatens his spontaneity and his freedom.

It is basic to the new pedagogy that we underline a need to understand logical structures of our own inventiveness as a bridge between the education of the individual and the objective culture of the societies that figure in the history of the world. Above all, the new teaching affirms the reasoning powers of the individual and tries to develop this individual to his fullest capacities. Mathematics is not primarily an educator's instrument for controlling nature or society nor a model for other sciences nor a universal science. It is merely an opportunity through which today's man can face the problems of his time and the threat of enslavement by machines, maintaining the initiative of free logical thought.

The discovery of a new pedagogy for mathematics has carried over into other fields. Physics and other natural sciences are also beginning to be taught with other criteria and new content, as are grammar, history, and philosophy. In spite of the force of these pedagogical currents, and notwithstanding their virtues for the history of modern man, we educators still offer considerable resistance to the new. Our profession, too, has its misfortunes and its defects. One of them is the conservative spirit which stubbornly permeates our activity. It will be difficult to banish the age-old dogma that academic mathematics is merely a collection of procedures employed to find the right answer for a mathematical operation. Traditional teaching only sought formal accuracy in procedure, the observance of strictly mechanical acts. Everywhere the ideal student is still the one who - in contradistinction to the Platonic idea - operates quickly, accurately and blindly with numbers through the decimal system of notation (the only system that he knows).

This pedagogy, so out of place for the new times, upsets the relationship between the child and the knowledge he requires and therefore between the person and the cultural significance of knowledge. The child becomes a counter who "does sums" with wholly practical ends, rather than a human being who sows in his mind the seeds of knowledge. So much mathematical talent has been frustrated by these procedures which separate the man from that which is truely most significant for him. All of this practice has undercurrents of ideas that have greatly influenced western culture during the last few centuries. For example, at the beginning of our era, it was thought that mathematics should simply be the "hand maiden of physics," or in other words, a practical instrument for controlling the world. And still, in our examinations, mathematics is measured by its pragmatic efficacy. Thereby, it has been losing its meaning for academic teaching. The false idea of Kant that all mathematical concepts have an intuitive counterpart in pure space or time can also be seen in the teaching of our science for a long time past, that is, the idea that all mathematics is bound up with the world of intuition and in the final analysis with practical and technical consequences.

Another basic aspect of a new pedagogical current in the teaching of mathematics is the teaching of apparently difficult concepts to children in the elementary school. It has been repeatedly shown that very young children may grasp these concepts with considerable pleasure. I recall once posing the following problems to a six year old child of normal intelligence: What numbers are at the same time less than five and more than ten? The quick answer cautiously expressed in childish language was "there aren't any." Those words led him by the hand of the discovery of the concept of the null set. The same problem was passed to grown-up persons, including engineers with traditional mathematics background. Everyone tried to find the requested numbers, incapable of logically solving a problem that was not at all difficult for a child. The child, with certainly little knowledge of inequations, was able to integrate his personal logic with the objective and universal logic of mathematics.

In summary, we may say that if technology and the drive for power with which man seeks to dominate nature and society has become so powerful in today's world, let us use the new pedagogy of mathematics to develop in man the initiative that will permit him to maintain his freedom of thought and life in the face of these powers. If technology and power have made our material circumstances easy, let us seek out the difficult paths that can be found before us if we wish to live in personal freedom with civic responsibility. The individual man educated to exercise the initiative of his thoughts and the spontaneity of his will along the rigorous paths of internal discipline is always a free agent. The crucial problem of present-day education is, in my opinion, to seek to reconcile general and specialized education.

spiritual initiative of the individual so that he may be able to serve society with the efficiency of a specialist and still for himself maintain the capacity to think in universal terms and to respect the norms of his society and the values of his culture. The revolution of the teaching o: mathematics promises us much as a path toward these objectives.

In the name of the government of Peru I am pleased to extend a cordial welcome to the scientists and teachers who have come to our country to consider problems which profound affect the destiny of humanity in our times - notwithstandin the asseverations of certain so-called practical minds. I have confidence in the capacity, the spirit, and the good will of all those gathered here on your ability to uproot prejudices and to universalize new methods to bring about new progress for modern education.

THE WORK OF THE CONFERENCE

Marshall H. Stone
Chairman, IACME

Mr. Minister, Colleagues, Ladies and Gentlemen: On behalf of the second Inter-American Conference on Mathematical Education, it is my privilege to thank our Honorary President most warmly for his gracious words of welcome and for his very penetrating analysis of some of the important problems confronted by all those interested in the future of mathematical education. I would like to add my own welcome to all the participants and observers who are gathered here in the opening session of this Conference and to wish you every success in your endeavors.

The remarks that follow are directed toward the work we hope to carry out during this conference. It is obvious that the problems of education are so broad, so extensive, and so difficult of practical solution, that we can touch here upon only a few of them. For that reason, the Organizing Committee, after careful consideration, chose a limited number of topics for study in this meeting. There are many more topics which need to be talked about but which will not be discussed here, although they may well be mentioned because they bear some clear relation to one or another of our principal themes. They are not to be the things given the first place in our present discussions, not because they lack importance but merely because we do not have the time to treat them adequately. For example, I think everyone realizes that the promotion of advanced study and research in the field of mathematics is an essential activity which has an important bearing on what is done in the pedagogical field. What we teach in our schools must be oriented according to the directions of progress in mathematics and the directions in which the applications of mathematics are being extended. We therefore have to turn for advice in curriculum planning to mathematicians, physicists, and other scientists who know at first hand where the frontiers of mathematics are being pushed forward and whither man is moving in his effort to develop his scientific and technical hold upon the physical universe. The fact that we are putting to one side so far as this conference is concerned these questions of advanced teaching and research does not mean that we ignore them nor that we eliminate them altogether from our thinking. It simply means that we want to turn to three other questions which seem to us particularly important, even urgent, at this particular time.

In the first place, it is natural that we wish to review what has taken place in the hemisphere since the first Inter-American Conference on Mathematical Education, held almost exactly five years ago, in Bogota, Colombia. We must now ask: "What had the report of that conference to do with what has taken place in the last five years? Have its recommendations had any influence at all? Have some of them proved to be less practical than we had supposed at the time when we formulated them? In which countries has progress been most marked? In through the medium of a number of reports, and the discussion of them, try to see what the impact of the first conference has been and what we have succeeded in accomplishing all over the hemisphere during that time.

This done, we wish to take up two particular themes which need to be stresses as exceptionally important at this juncture - themes which are important for every country represented here even though they may appear under quite different aspects in different regions.

First, we wish to take up the very difficult problem posed by the students' passage from the secondary school into the university. This means looking at the student from two points of view. Naturally, we must ask "What does the student learn in the secondary school? What point does he actually reach there in his mathematical education?" Then we must look at him as a matriculant in a university where he will have to adapt himself to unfamiliar teaching practices, stiffer competition from his classmates, and more exigent standards of performance. The secondary school teacher and the university professor or administrator look at the student from rather different points of view. In consequence the student may - and too often does - discover that his preparation for the university has been inadequate and that he is suddenly called upon to rise to higher level of achievement. Everyone who thinks back for a moment to his own student days realizes exactly what this has meant in his own experience. The student often suffers acutely from the feeling of being lost in the atmosphere of the university and of being forced with little or no warning to learn in a short time and with little or no help from his professors many things which he should have learned already in the secondary school. If we consider the matter from our own special point of view as university professors, we have to recognize that rather generally we expect our students to be very well prepared and altogether ready for the work we give them at our level. What we often fail to do is to consider that we may possibly be asking too much of them too soon.

The problem of transition is more or less critical according to the circumstances existing in different countries, and the solutions that have to be sought must depend very much upon these circumstances. We should not try to employ universal methods or measures which are clearly inapplicable in dissimilar situations. What we want to do here is to discuss this perennial question from each of the national view-points as a basis for reaching a general intellectual grasp of its nature and then to see what the possible solutions to this very difficult human problem may be.

We must realize that in this situation we are face to face with a grave dilemma. On the one hand, insistence upon a high standard in the universities lead either to severe restrictions upon admissions or to a strict policy of failing those who may be admitted despite inadequate preparation or lack of qualifications. On the other hand, we know that in taking up careers in those professinnal lines essential to the development of a flourishing economy. To embrace either horn of this dilemma would be to court disaster. We are therefore forced to seek a solution in compromise, accepting some lowering of standards in order to increase the number of students entering the university and going on to a satisfactory conclusion of their studies. However, this solution cannot be regarded as one which can be accepted for the long run or as one which can be considered as adequate unless the university offers effective assistance to the serious but ill-prepared student. If standards are lowered other than purely provisionally, the consequence will be that the schools, no longer challenged by the difficulties of admission to the universities, will lower their own standards and send out their graduates more poorly prepared than ever. Thus any temporary lowering of university standards must be accompanied by measures intended eventually to bring those standards up again as rapidly as the inherent difficulties in the transition from school to university can be overcome. Concessions made to the poorly prepared student must be coupled to an effort on the part of the university to emphasize the importance of good teaching and well-planned and coordinated courses in the first two years. Such concessions, furthermore, must be continued only so long as it takes the schools to meet their own responsibilities for providing better preparation. If schools and universities can approach the solution of the problems posed by the it should be possible to eliminate the temporary concessions and restore standards after ten years or so of hard work.

The second problem of immediate interest which we intend to discuss in this meeting is the problem that everyone now recognizes as the master key to any substantial and permanent reform in the teaching of mathematics. I refer to the preparati of teachers for service in the primary and secondary schools. What can be done toward improving the schools depends in the last analysis upon the attitude and the capacities of the teach themselves. It does little good for critics or the public to exhort teachers to make this change or that unless the teachers believe that the tasks proposed are the right tasks. If the teacher thinks that mathematicians or educational theorists are asking that the wrong kind of mathematics be taught, he may yield to the imposition of one new program after the other but he will teach none of them with enthusiasm and his lack of enthusiasm will inevitably be communicated to his students. The best of programs can well fail in these conditions. It is just on this point that summer and academic-year institutes have had a beneficial influence, because wherever successful institutes have been organized the obvious practical result has been that many teachers have revised their ideas about the teaching of mathematics, about the topics in mathematics which should be taught, and the ways in which those topics should be presented. They have acquired enthusiasm for moving ahead in the first steps of a reform which they themselves have come to enthusiasm alone is an insufficient foundation upon base the reform. The teacher who desires to do a job in presenting mathematics and helping his students stand mathematics is going to be powerless if his asm is not backed up by a sound and even deeper knowl- mathematics. Unfortunately, there have been far too enters where the preparation of teachers has been guided hilosophy of ignoring training in subject matter in o concentrate on pedagogical and psychological prepa- of a theoretical character. The result is that our ams for the training of teachers stand in urgent need of ion, reform, and even revolution, just as much as do those schools and the first years of the university. The time now come when we can no longer postpone giving this ent question our full attention. Together we must begin nquire anew what it is that the teacher needs to know in r to do a first-class job in teaching mathematics at the ondary level. We have to provide this teacher with training n mathematics, and also in pedagogy, which will make him a etter and more enthusiastic teacher. Above all, we must realize hat it is not enough to do this now, once and for all. As we ook ahead we see that the kind of change we must try to make w is only the first of an unending series of changes which ll be needed in order to keep abreast of developments in thematics and its applications. Because of scientific and thematical progress we are going to be compelled continually revise our programs for the teaching of mathematics; and s means that those who teach mathematics at whatever level, be it in primary school, secondary school university, or post graduate institution, must all have flexible minds and the ability to increase their knowledge and to take advantage of every important pedagogical innovation. For instance, we cannot expect that, once we have taught a new generation of teachers the geometry which we believe should be included in the secondary school program, school geometry then will necessarily remain the same for all future generations. We must not leave the teacher with the belief that he can safely ignore advances which may be made in geometry after his formal preparation in teachers' college or normal school has been completed. In short, the teacher of the future must be able, willing, and eager to learn what is going on in the different branches of mathematics and to keep in continous contact with advances in the art of teaching.

Here then are the three themes on which we have decided to concentrate at this gathering: an analysis of progress, a discussion of the typical transition from school to university and the preparation of teachers. Our discussions here must point toward the future. It is not enough to have a challenging and exciting discussion of these problems. When we have closed our sessions we must have arrived at some sort of practical conclusions, or at least at some well-defined point of view which will lead us to effective action in our respective countries. Therefore, we have to organize our discussions and bring them to a point where we can summarize them and make them available not only for all of us who are here but also for the many others who would be interested in knowing our views on these subjects. Our meeting is necessarily a small one - we could not have worked effectively in a very large meeting even if we had had the means for arranging one - but the people with whom we should have contact are extremely numerous and are spread over the whole vast area of the western hemisphere. We want to reach all these people in one way or another with our conclusions and our report, and we want to achieve practical results growing out of our deliberations here.

In my opinion there is only one way to carry out a thorough reform of mathematical education at the national level. First of all, one must identify the problems that present themselves in that particular country. Next, one has to examine these problems and to compare them with similar problems encountered in other countries in order to learn from the wisdom or the folly of one's peers and one's colleagues in every part of the world. As a result of this study it should then be possible to formulate some idealistic, perhaps even unattainable, aims as the goal for action. It is only with a clear objective in view that one can initiate intelligently or execute effectively any kind of practical steps or action. If we have no idea of whither we wish to go, then the successive approximations we believe we are making may turn out to be divergent; and in the end it may turn out that we have only wasted both time and effort. But if we place before ourselves a clearly defined ideal, even though we may eventually have to redefine it in the light of experience and expanding knowledge, then we shall be able to see what practical action is necessary to realize that ideal. In case the objective is one which clearly cannot be accomplish in less than five years, or ten, or eventually in twenty, we can nevertheless plan the sequence of practical steps by which we may move slowly but surely towards our goal.

Too often people have started a reform by saying, "We shall do this now, in a hurry, to change the last two years of high school mathematics," or, "We are going to do this right away in the first two years of the university" - and "this" is indeed done, but in such a way that is has no relation to anything else which is being done or needs to be done. If there are serious difficulties which lead to changes in the original aim, then we may not be able to measure what has been accomplished against any stated goals nor to see whether we have realized a substantial achievement. In this type of reform effort it is easy to be discouraged or to remain content with only a minor achievement, simply because there is no over-all objective. Or if further steps are undertaken they may not lead to more than an aimless jumping from one experiment to another.

Thus having accepted this point of view about the need for careful planning for the future, I feel that one of the most important things we can do here is to delineate a picture of what we think should be done ideally about the main problem we shall discuss and to provide a means of expressing our conclusions convincingly and giving them a practical effect.

It has therefore been one of my deepest concerns to arrange for the choice of two committees which will deal with these matters of the future. Thus, I conclude my remarks by reporting the agreement we reached yesterday afternnon in the Organizing Committee for setting up these two committees. One committee is to deal with the question of the conclusions and recommendations of this conference. Professor Pereira Gomes has consented to serve as chairman of this committee, and I shall ask him to arrange for an early meeting of his group.

The other committee must deal with a matter of equal importance, it seems to me, and one which is of a highly practical nature. This is the problem of how the kind of work that we started to do in Bogota and that we are continuing this week in Lima, can go forward in the future. At Bogota it was decided to elect a committee of five members to attempt to implement the recommendations of the conference. The committee was duly elected, I was chosen a member of it, and the committee itself insisted that I serve as its president. This I have done to the best of my ability, though I have frequently been overwhelmed with pessimism occasioned by the difficulty and the magnitude of the tasks undertaken by the committee. The committee was clearly not quite large enough to give adequate representation to the diverse interests involved; but by using its authority to reform itself the committee was eventually able to add a sixth member representing Mexico, a country which had not previously had a proper place in our activities.

The committee then had to find some way of securing a degree of permanence and of laying out a suitable mode of operating. At first the committee floated in the air with n official point of attachment and very little financial supp We explored various ways of transforming ourselves into a somewhat more permanent international body. This was a rath difficult thing to do because each existing organization of an international character to which we might have become attached has its own policies and its own ways of working. It seemed to us that inasmuch as we were an elected committe we could not simply subordinate ourselves to some other permanent organization, but that instead we had to remain an autonomous group at least until such time as there could be another meeting and some other arrangement could be establis Eventually the committee associated itself in an autonomous and very free relationship with the International Commission on Mathematical Instruction (ICMI) of the International Mathematical Union. If anyone asks what the offical status and the authority of the Committee are, the answer now is as follows: The conference at Bogota was called and organized by the international Commission on Mathematical Instruction in the International Mathematical Union; that conference elected the Inter-American Committee on Mathematical Education (IACME with authority to organize and modify itself and the duty of implementing so far as possible the recommendations of the conference; and IACME is now an autonomous regional body affiliated to ICMI in accordance with the [illegible]

That then is where we stand today. We do have an official point of attachement or affiliation, and we have practically no money. The practical questions which have to be answered are: first, what kind of committee should we now form and how can we choose the membership of that committee by election at the close of this meeting; and second, how can this committee be assured of enough financial support that it will be able to meet occasionally and to plan not only for reasonably frequent conferences like those of Bogota and Lima, but also for some kind of truly effective cooperation with all the splendid national efforts being directed at the reform of mathematics teaching.

One reason that we are particularly grateful for the presence of the honorable Minister of Education of the Republic of Peru and for the interest which has been shown this conference by his Excellency the Minister of Education the Republic of Chile, as well as for the interest shown by everal other Ministers in various ways, is that we are now to me extent recognized as an effective organization, however eble at the moment, and that we can perhaps count on a tter understanding and some sort of support from the nistries of Education in the western hemisphere. Actually, I lieve, we have already received an official recognition at meeting of Ministers of Education held in Buenos Aires t June. According to a quotation from the official record, given me by our colleague Mr. Völker from Buenos Aires, this ting of Ministers of Public Education adopted a resolution h in effect encourages the Ministries to consult with this committee and similar international bodies with regard to education at all levels, primary, secondary and university. If this resolution is taken seriously and developed into a closer relationship between our committee and the public bodies concerned with education in the different countries of the hemisphere, perhaps we can go forward in the realm of practical action. In short, I consider that second working committee to be set up has equal importance with the first, though the charge to it is of an altogether different nature. Its first task, of course, is to choose a good membership to take over from those who have worked hard during the past five years and have after some difficulties succeeded in organizing this conference. I have asked Professor Laguardia to be the chairman of this committee, and he has kindly consented to do so.

With these words I now invite you to begin your work. I thank all those who have shown their lively interest in this occasion by being here this morning and listening to the interesting discourse of our Honorary President and to my rather too long and necessarily practical remarks.

PART II

THE ADDRESSES

A. PROBLEMS IN MATHEMATICAL EDUCATION IN LATIN AMERICA

SOME OBSERVATIONS ON THE DEVELOPMENT OF MATHEMATICS IN LATIN AMERICA

Raphael Laguardia

In an article that appeared recently in the review La Educacion, 37-38, 1965, I consider the development of mathematics research in Latin America and pointed out some of the obstacles before it. Elsewhere, in the bulletin of the Uruguayan Association for Progress of Science, I describe the conditions which I see, in the light of my experience, as needin to be taken into account in organizing our mathematics institute to function efficiently. Finally, in the Uruguayan report which I have presented to this conference, I have taken up the problem relating to instruction and research in my country. This backgro as well as the talks of Professors Tola and Santalo who have preceded me suggest to me that I may focus on some points which I think particularly important with respect to Latin America. Some of my observations are well known and will seem banal in advanced circles, but I feel the need to reiterate them since our traditions, organizational weaknesses and vested interests perpetuate a very unhappy situation in Latin America.

The passing of the century has seen scientific and technological advances of an intensity, speed and extension with precedent. In particular, mathematics has experienced an impressive increase in the number of publications and enormous quantitative development of research and the proliferation of new branches. Furthermore, thanks to axioms which facilitate the construction of abstract models, mathematics has invaded new areas: Today it is not only applied, as heretofore, to the sciences concerned with inanimate nature; now it has tasted the biological, economical, and social sciences to such a degree that all scientific and technological development of our society, all material well-being present and future depends fundamentally on the development of mathematics (that is, logical thought).

It is understandable, then, that we have an urgent imperative in Latin America to promote the development of mathematics and the other basic sciences. It is worth while then to pause and reflect upon some of the obstacles and defects that hamper growth and encourage the widening of the gap that separates us from the advanced nations.

a) In the first place, except for two or three countries, illiteracy reigns. Large sectors of the population lack even an elementary education, receive no instruction or drop out prematurely from elementary school. Dropouts are also common in high schools and even among those few privileged students who reach university level.

Given the percentage of intellectually gifted people as being the same in the various classes of the population, illiteracy constitutes an inexcusable waste of the most valuable resource available to a nation today, the resource of talent.

the necessity which all states should feel of giving priority to the extension of elementary instruction.

b) With respect to high school instruction, the picture is darkened by the rapid growth of the population which makes it necessary to use poorly qualified or inadequately educated teachers. Even in Uruguay, where there is little population increase, the same difficulty is found because, happily, high school instruction is tending to be extended to the entire count

The available resources of Latin America do not permit us to think seriously, at least for the moment, of massive reeducat of in-service teachers as has been attempted in other places. Ev short intensive improvement courses, whether called summer cours vacation courses, project 212 or whatever, usually reach a comparatively small number of educators who generally acquire only an ephemeral cultural veneer and a diploma which sometimes helps them advance in their careers. Such courses can only be effective if they are incorporated into a carefully planned program of permanent activity that develops in progressive stages and is implemented by qualified groups of researchers and teachers from the universities, the normal schools, and the high schools. Among these directors there must be some one well versed in the practical aspects of the psychology of young people and the pedagogy of mathematics.

c) I believe that the reforms, at least in my country, must begin at the second phase of intermediate instruction, where there are fewer difficulties because the student body is smaller and the faculty is better prepared. Changes will have to be gradual; they will have to be announced well in advance; and some permanent orientation body will have to make contact with in-service teachers and supply them regularly with the best theoretical and practical information possible. In this way a group of leaders will be formed through which the reform can be extended - with the same precautions - to the first phase of intermediate instruction. From there a new advance will be made in the second cycle; and thus alternately back and forth.

d) The intervention of the university in the reform is an indispensable requirement unanimously recognized in the international conferences, particularly in the one in Bogota. But this must not distract intermediate instruction from its specific purpose, which is not to transmit to the student a mass of dead or necessarily ephemeral knowledge but rather to make him participate actively in the dynamic process of the evolution of knowledge, helping him - at an age when he is particularly receptive, though unstable, sensitive and emotional -- to develop his creative potential, find a place for himself in society, and rticipate consciously in its history.

e) Since I have mentioned researchers, I should like to hasize their importance in teaching. I refer not only to those earchers in the area of instruction but also mathematics researchers. In contrast to the scholar -- with whom I am actually at no very great odds but who constitutes among us a vestige of the cultural inheritance from Spain -- the researchers, because of their work and their professional habits, bring to instruction exemplary models of tenacity, imagination, the spirit of initiative, ability and facility for facing new situations, acquiring and transmitting new ideas and knowledge, adopting new points of view and teaching with energy and interest. This has a very personal educative effect on students.

In addition to the great or small contributions that the researcher may have made to scientific and technological progress, it is enough to point out the aspects to which I have just referred to understand that the cultural rise of Latin America and its economic liberation are closely linked to the intense development of research and the determined attitude it imparts. One cannot fail to mention my astonishment upon visiting the University of Concepcion and discovering that the report prepared by the various foreign experts on the creation of the central institutes of basic sciences did not mention at all the development of mathematical research, as if that could not or should not be developed at the same time and level as higher instruction. If some message is to be gleaned from my words, it is: We must fight tenaciously against the mistaken and negative idea, unfortunately established among us and promoted by some people abroad, that we must develop our intermediate instruction and then take ...

of research. By that path we would only increase our dependency and our backwardness. On the contrary, we must strengthen our best centers of scientific research and higher instruction and call upon them to collaborate closely with intermediate instruction and seek out mathematical talent early, cultivate it, send our most talented young people to good foreign centers to finish their studies and take what measures are necessary to have them return and stay among us.

With this (and in closing) I touch upon a point to which I referred in the aforementioned article in the review "La Educacion." There I said, "In the current stage of the development of science and technology discoveries and inventions occur one after another and the time between their discovery and their practical application grows shorter. If we bear in mind that the maturation of talent continues to need long years it can be understood that demand has come to rule and that the search for, development, profit or exploitation of this wealth has an interest and urgency today that transcends national and regional boundaries and often assumes very seductive forms. It is in the greatest collective interest of Latin America to find effective means of discovering scientific talent in its most useful and highest form, aptitude for research, that is to say, the capacity to pose, face and solve problems, to cultivate this talent and retain it. The unhappy tendency to accept the division of countries into those who create culture and those who consume it can only widen the gaps that separate them."

THE PROBLEMS OF MATHEMATICS REFORM IN LATIN-AMERICA WITH REFERENCE TO TEACHERS AND PROGRAMS

Louis A. Santalo (Buenos Aires)

It may be said that as a result of the meetings and conference on mathematics education that have taken place during recent years in various parts of the world and also due to the opinions express by many distiguished mathematicians in articles and lectures, the is a rather concrete understanding of the kind of mathematics tha should be taught at the respective levels. At least for high scho and basic university levels there are sufficient programs and experiments to provide reference examples and basis for discussio when we deal with the problem of introducing modern mathematics in teaching.

There is no doubt that to achieve a complete evaluation of the new programs and methods we should have to have experience including the three phases (elementary, intermediate, and higher), free from incommodious intervention and interference. Nevertheless at least as far as we know and notwithstanding the valuable experiments of Papy and the profound psychological studies of Piag there is still no general concensus regarding the program for modern mathematics at the elementary level. For this reason, in th following discussion of the problems of mathematics reform in Latin-America, we shall be referring exclusively to those of the intermediate or basic university level. And evenso, greater emphasis will be placed on the secondary level, since it is there that the problems are most acute in terms of both intensity and volume.

At the basic university level, in places where there are schools of sciences, it may be said that the reform has been accomplished almost everywhere and almost in its entirety. The reform is proceeding more slowly in the schools of engineering and in technical preparatory schools; but as a matter of urgent necessity it is being carried out, since without it, it is impossible to read any of the new books being published on any technical subject using mathematics. The technical specialists themselves, then, are the people who are most interested in attuning instruction to current bibliography.

On the other hand, at the intermediate level, much remains to be done. Travel in many countries indicates that there is widespread concern for progress. As is always the case with any innovation, there are obstacles to overcome and problems to solve all the time. It is the object of this talk to analyze these problems, at least those which seem to be the most widespread and common to the majority of Latin-American countries. At the same time, we shall suggest some possible solutions. Some of these problems could be foreseen ahead of time; others, have arisen along the way. In both cases, they can now be defined more clearly than would have been possible a few years ago, thanks to the work that has been done and the direct or indirect though these problems are intimately linked among themselves, we can follow the Cartesian recommendation and divide them as much as possible, to feel them more strongly and be able to deal with them better.

Problem 1 - To Convince the Teachers

The first problem is convincing the in-service teachers of the need for reform and the possibility of implementing it. The solution to this problem is basic and must precede any other measures, since without prior convincing of the faculty which will have to do the implementing, any reform imposed from without would be empty and of dubious results. For demonstrating the need of the reform we have at our disposal the following arguments: a) the recommendations of the various meetings and international congresses on this problem as well as the most outstanding contemporary mathematicians; b) the fact that the great majority of current books in which higher level mathematics is applied to some specialty (economics, medicine, architecture, industrial organization, operational research,...) utilize modern mathematics; c) the great progress made by contemporary technology both in extending possibilities and in accuracy of results, a technology at the base of which modern mathematics occupies a primary sustaining position; d) the fact that in the traditional programs there is no mathematical concept less than one century old, which puts us out of place with time, to an indefensible degree; if we are to prepare citizens for the second half of the twentieth century, it seems natural to do it with tools characteristic of our time.

With these arguments, repeated and illustrated, it is not difficult to show the need for reform.

There remains the second part, the demonstration of the possibility of reform. The natural reaction of the teacher, once he is totally or partially convinced of the need and even the urgency of the reform is to say, "All that is very well, but the student will not understand it." To overcome this objection and convince him that modern mathematics, like languages, is learned more easily by young students than by older students, the best argument is the experiments that have already been carried out. This should be sought as close by as possible, in neighboring countries or when possible in the same country. In Argentina, in this regard, it has been very helpful to have teachers in charge of pilot experiments in certain schools visit other parts of the country to let people see the exercise notebooks and classroom tests of their students. This stimulates some teachers who are initially skeptical to decide to try the experiment of giving a course with the new syllabi. The result leaves them not only convinced but among the most enthusiastic promoters of the programs.

One final and difficult stage remains in the problem of convincing teachers, the task of convincing them that a large part of the mathematics they have been teaching for years is no longer useful and must be eliminated from the syllabi. As a matter of fact, the teachers generally resist the elimination of certain topics much more than the incorporation of other new ones. Inertia and tradition pressure strongly for continued teaching of what seemed indispensable for so many years; and the elimination of these topics seems truly catastrophic to many teachers.

We shall cite two possible arguments for overcoming this resistance: a) inviting the teachers who insist on the necessity of certain topics that we now propose to eliminate to inquire among persons they know who learned these things in school and ask how many times they have used this information in their later lives; b) once the method suggested has invalidated the idea of practical utility of the aforementioned topics, we can examine their educational value by analyzing the amount of reasoning as against the amount of routine in those topics; we then may compare this with some modern topic which we propose to introduce. The result will certainly favor the modern topic.

Problem II - Convincing the Parents of the Students

Not only must teachers be convinced; the parents of students must also be convinced, since their collective influence on the school and on the administrative bodies can not reasonably be ignored. Actually this problem could be stated in a more general way as being the need to convince the environment or public opini

With respect to this, I should like to refer to a concrete case which is very illustrative of the reaction of public opinion to instruction in modern mathematics. Around the middle of the school year, I met a friend of mine who is an engineer and has a fourteen-year old son who was in the second course in mathematics of an intermediate level school with the new programs. When I asked him how his son was doing, he replied, "Right now he is doing well and he is quite happy, but I am worried because we are half way through the year and the teacher has not yet begun mathematics." I checked into the program and realized that from the father's point of view all that the teacher had been presenting up to then (sets, relationships, functions) was not mathematics. For him, because of what was drilled into him in his day, mathematics was exclusively the manipulation of complicated polynomials or the reduction of roots to a common index. I had to convince him - and I probably only half succeeded - that mathematics changes with the times, just as the materials for the construction of buildings do or just as remedies for disease do.

In any case, to overcome this resistance of the environment to modern mathematics, it is well to bear in mind that mathematics, besides being an educative science with its own content is also a science that is instrumental in almost all the others; so its applications should not be ignored. It is a good idea to show applications of modern concepts to physics, biology, social sciences, economics, etc., whenever it is feasible (which means quite frequently). Nor must one forget the basic operations that the student has learned in the elementary school and must continue to practice, perfecting and extending them with the since the use of mathematics as a calculating tool is not excluded from modern tenets and is the part most valued by public opinion.

Problem III - The Preparation of Teachers and Textbooks for Stude

With the problem of convincing teachers there arises the problem of their preparation and modernization, which can be viewed in two aspects:

a) The Training of Future Teachers

This problem admits a relatively easy solution. We need but introduce modern mathematics in the teacher-training institutes and schools where intermediate-level teachers are prepared. This is not too difficult since at the university level there are many textbooks and teachers usually have more time to modify their syllabi in accordance with new developments.

b) Modernization of In-Service Teachers

This is probably the most serious problem confronted by the mathematics reform. The difficulty stems from various factors, the principal ones being the following:

1. The great number of teachers, many of whom are in small cities without centers of higher learning or direct channels of information. These teachers find themselves isolated, with nowhere to go for orientation.

2. The speed with which the student population of secondary schools is growing. In many countries this has made it necessary to employ teachers without proper training. In addition, the large number of students makes crowded classes inevitable and hard to

handle, particularly on the basis of continual professor-student exchange as is assumed with the new mathematics.

3. The overloaded work-schedules of each teacher. The system of pay by weekly class hours (and salaries always being low) obliges the teacher either to teach long hours or devote himself to other activities. Where the teacher is overloaded we can not demand of him the appropriate attention and great effort required by the initial implementation of the new programs. Even supposing that the teacher is duly supplied with the necessary information and is quite willing, he cannot devote the necessary care to starting new programs which need much preparation, careful daily observation to analyze the reactions of students, and a careful search for unusual exercises in traditional textbooks; for we are speaking of a teacher with thirty or more weekly hours to teach. The ideal is to work toward a full-time teacher with a sensible number of class hours and assured stability, but this seems still to be a distant goal in our countries.

To combat the first two aspects of the problem, trial has been made of the method of organizing intensive workshops to inform teachers of the new developments in mathematics and discuss together with them the new programs. This method is locally effective but on a world-wide plane it is very slow, even with the chain-reaction system of having the participants in one series of courses later organize others to instruct their colleagues at home in the areas they have studied. This system has some usefulness but is not without the dangers involved in

Another method employed is to supply teachers with guide books, model syllabi or monographs on special topics which teach may adapt to the level of their students and develop in their ow way. This method is not as effective as one may think since the average teacher needs more than guide books or synthetic monographs; isolation and routine after a few years makes it difficult for him to undertake the work of adapting the material

We believe that the only method will have to be the publica of textbooks for students, which at the same time will show the teacher in detail the kind of instruction that is desired. These texts may be supplemented by teacher manuals with additional information and advice for focusing the expositions of different topics; but what is important is that they be texts that are thought out and written for the students. The teacher who is used to following a traditional text will exchange it for the new one, and even though it is uncomfortable for him at first to adapt himself to the approach, two or three repetitions should serve to clarify the sense of the new mathematics and assure the reform.

Since we are convinced that this is the best and perhaps the only way to achieve some speed and probability of success in introducing modern mathematics at the intermediate level of the schools in Latin-American countries, we should like to suggest that this Conference on Mathematics Education adopt as one of its recommendations the following: "To interest international institutions in the reform of mathematics instruction at the secondary level so that they may make available the necessary funds for the publication of one or several series of textbooks adapted to the different years of the intermediate phase. Once these texts have been published and circulated, should the demand be sufficient, special contracts could be made with publishing houses in various countries interested in publishing later editions of the book at a commercial but not abusive price."

In this way the institution would initially be only the promotor, even possibly recuperating its invested capital eventually. We say "one or various series" because it would be helpful to have several kinds of exposition and points of view, within the general lines which would be established by an advisory council to insure that the textbooks are modern, suitable to the objectives, and an organic part of the respective series. Certainly these texts would not be perfect and would immediately generate criticism and debate, but the initial step would have been taken and would serve as a starting point for new textbooks written by private publishers and teachers whose mutual competence would guarantee continual progress and improvement.

Problem IV - The Inflexibility of Regulations

A problem which makes gradual implementation of the reform in Latin-America difficult is the inflexibility of regulations. Schools are generally dependencies of the Ministry of Education or a similar organism which insists that all schools meet the same norms and use the same programs. The reform is not accomplished because of excessive caution or it is implemented all at once and equally in all schools. It is difficult to secure authorization for some teachers to introduce modifications in instruction or to try out new points of view. There is too much fear of the disorder which supposedly might ensue and the difficulties which students might have should they have to change schools or teachers later.

This makes experimentation and try-outs difficult. While it is true that all innovation must be controlled so as not to exceed certain prudent limits, it must be kept in mind that some experimentation and a margin of confidence in the teacher who wishes to try out the recommendations of his colleagues or responsible institutions of other regions or countries is useful and almost necessary. In Argentina we have made considerable progress in this liberalization; some personal initiative has been permitted and has shown - and we hope will continue to show - positive results.

Finally, there is another problem, pleasanter and less important than the previous ones but which I should like to point out; it is the problem of the overly enthusiastic teacher. Actually, new ideas always have their fanatics who are anxious for novelty or their revolutionaries who interpret everything in their own way and carried forward by their enthusiasm and with the best of intentions do more harm than good for the idea they try to defend.

In the reform of mathematics teaching, the case does also arise of the teacher whose enthusiasm exceeds his training and who tries to make his own course out of the most superficial and incidental sections of the books and recommendations. His course is generally filled with trivialities if not conceptual errors and always general confusion. This produces a natural annoyance in the environment with respect to the reform whose objectives he claims to be following. This is another reason for insisting on the need for many and varied textbooks. The most appropriate one may be selected for the environment and the personal taste of the teacher; but there will be a guarantee of seriousness and correctness.

As a result of these observations we may say that though there is no shortage of problems, and the difficulties are far from negligible, they are now quite clarified and in many ways and aspects on the way to forthright solution. From this we may gather faith and the sentiment that the reform in mathematics instruction will soon be a fact in all of Latin-America.

PROBLEMS IN DEVELOPING MATHEMATICS RESEARCH IN LATIN AMERICA

Jose Tola P. Lima, Peru

Again as for the first time in Bogota in December 1961, we meet to take up everything connected with mathematics instruction in the countries of Americas. Again our goal is to bring new ideas and methods essentially designed to modify the content of mathematics instruction at all levels. And we continue with this task because we believe that as yet mathemati instruction in our countries is not organized as it should be fo its greatest benefit. Actually, this instruction is largely governed by antiquated norms whose harmful characteristics have been pointed out many times without their having to be repeated here. They are as useless for intellectual formation as for scientific and technological training which are so urgently needed.

Again as in Bogota, we must consider the present situation, appraise and evaluate the progress that has been made, and outline plans for future development; we must again compare our experiences, heed the teaching of those who can factually demonstrate the excellence of the methods they are employing, and seek the most effective paths to the desired results.

The purpose of my talk is to refer to some aspects of development in mathematics in Latin America. More concretely, countries in which reform activities have not been done or have begun in a poor manner. But first I must place the question I am going to treat in the context of the agenda of this conference; although the main theme of the Bogota Conference was mathematics instruction at the high school level and higher, still the discussions held at that time pointed insistently to the importance of mathematical research in any central raising of mathematical standards. And this is so much so, that among the recommendations of the conference competent authority insists on 'the stimulation of research as the core of scientific and technical progress and as an element of inspiration and instruction.'

Between the Bogota Conference and today a number of international meetings have insistently repeated the same affirmation regarding the basic role of scientific and technical research for development. Nevertheless, little had been said about the tactics needed to establish research where it has not yet begun or where it is in an incipient stage. It can be said that this particular research situation has not changed from that which it was in 1961. I believe that this situation has been and continues to be responsible for not having enough mathematicians and consequently the resources for accomplishing the task of reform in instruction.

Further, this problem has grown worse in recent years due to the growing demand in our countries for qualified mathematicians particularly due to the creation of a large number of new institutions of higher learning.

Before touching upon research itself I should like to offer some observations from experience of these recent years hoping to thus clarify my point of view.

It is an undeniable fact that in matters of education and especially mathematics education our countries represent a great variety of situations each of which requires particular attention and a primary consideration by local efforts. Recommendations such as this one are valuable and important only in so far as they can be applied to a variety of situations in ways found to be most convenient and adequate in each country. But the specific application to particular countries require the existence of competent local groups interested in the work, for whom these recommendations may be presented not only as an authorized form and procedure, but also as a valuable support to strengthen their activities. Such activities, designed to reform present systems, mean the abandonment of antiquated concepts, the replacement of outmoded programs, the re-education of present-day teachers, and the remolding of future teaching degree programs along radically new lines and finally the complete remodeling of school texts. These activities often require the intervention of the authorities.

Permit me then to focus upon what I call the local groups and which I find of capital importance in the task of reforming mathematical education. It seems obvious to me that where such groups do not exist there is no hope of accomplishing the task and that as this group has numbers, capacity and influence to that degree be able to bring greater efficiency, speed and extension ask. In the most desirable case the reform will include vels and satisfy all demands on the role of mathematics on both from the purely intellectual viewpoint and as ortant factor in scientific development.

should like to be more specific as to what I understand group in this context. I mean a group of people in a on to accomplish improvement in mathematics education the elaboration of projects and the effective execution se projects. My purpose at this time shows the fundamental t can be played in this group by the university professors; mathematicians employed by the university.

believe it is a fact observed in more than one country e initiative for reform of the content as well as the ology of mathematics instruction originates with the uni- y mathematics professors

Another observation that can be made is that which has been ssary to include in the concrete plans for reform some pro- ions for working with inservice teachers as well as provisions changing the teacher training systems. In both these projects great obstacle which has substantially hampered the effort and occasionally reduced it to an isolated episode of questionable onsequence has been the shortage of competent mathematicians to direct and accomplish the task satisfactorily. The aforementioned reasons and others too numerous to mention seem to me to leave the conclusion that one of the most important phases of the training of mathematicians, that is, teachers with a solid scientific knowledge not only for university teaching but also for planning, directing, and executing many of the most important tasks that must be done to carry forward the reform.

Having simplified, as much as I am able, the prefatory considerations, I now proceed to show that the preparation of mathematicians must be viewed as a high priority task even if we ignore the many reasons linked to our concepts of culture and development and heed the strictly educational reasons which have brought us together here.

In any case, I wish to add to what I have said that my own experience and the experience of all who have cooperated with me in the work done to date has led us to this conclusion, from which, in turn, we have deduced new formulas for action designed to intensify the education of mathematicians. These new programs, far from paralyzing the most immediate and direct plans for reform of instruction, are making it possible for us to activate them in a manner appropriate to the problems that must be solved. The new concrete formulas to which I refer have included the establishment of the Regional School of Mathematics which provides an accelerated education for teachers of higher mathematics who then serve in the Normal Schools and the early years of the university. The school has shown excellent results in the one year that it has been functioning. There has also been intensification of the efforts to train high-level mathematics teachers

This task has only recently begun but will be the basis of all future programs, to which later important programs will have to be subordinated, since we feel that the improvement of mathematics education (and to some degree the development of the country, in the broadest sense of the word) depend on this preparation of high-level mathematicians.

It is therefore quite proper that in this conference we concern ourselves with the education of mathematicians or the development of mathematics research, an activity which is inseparably linked to advanced education - as was declared in 1961 and which I am sure you all still feel. If we wish to outline a plan to develop research without leaving efforts to chance and improvisation, it is indispensable to determine, with great clarity, the precise objective. At least in the initial phase, in which most of our countries are at present, we feel the objective must be the formation of active university centers or at least centers closely related to the university.

Actually no great effort of imagination is required to describe these active centers, since a few excellent models exist in some Latin American countries. Nevertheless, we shall try to establish the principal characteristics of such centers in order to analyze the problems which may arise in establishing them and in their development. Thus we shall be able to approach the formulation of concrete plans without being restricted to the statement of a number of aspirations which will seem difficult or impossible to a[illegible]

Essentially the kind of active center I have in mind is made up of a permanent group of men whose scientific training qualifies them for research and for training teachers and researchers among other things. The characteristics of permanence on the one hand and contributions to teaching on the other, suppose close links between these active centers and the university. I do not think it necessary to go into detail here to justify the convenience and even the necessity of this link, to which I shall refer later. It is a fact that has been demonstrated repeatedly by experience that the establishment of an active group such as I have briefly described meets with great difficulties from its inception. A plan designed to permanently establish such centers must take into account these difficulties and include means to combat them. The establishment of a center implies carrying out certain programs which probably should be implemented simultaneously. They do not all have the same importance or the same urgency, apparently; yet they are all essential if we consider the matter from an overall point of view.

Above all it is necessary to admit that for the starting of programs such as those I am discussing, we must assume the prior existence of a group of persons interested in carrying them forward and equipped to utilize the means that may be placed in their hands for that purpose. With this prefatory clarification I shall refer specifically to those programs in their approximate order of importance, although, I have already stated they should be implemented simultaneously.

It is obvious that the primary element in an active group of mathematicians such as we desire is the man power element. Therefore, first in our purposes must be the getting together of the staff of teachers and researchers. In a sense this part of the project is the only one that need be taken into account, since all the others can be considered supplementary to it. But we prefer to consider it here as one program of the complete project.

A practical point which comes to mind immediately is the idea of contracting professors from foreign universities. When such procedure is possible it is undoubtedly efficient and rapid for getting serious training and research activity under way. Nevertheless it must be pointed out that we can only say that the group is effectively established if its members are closely tied to their environment. By this we mean that the project should be implemented in such a way that it fulfills all the goals that justify its establishment. Among these is the objective of extending the influence of the group to other activities; also that there be some guarantee of continuity of effort in training and research; all of which implies personnel remaining at the center for long periods of time. For this reason a group basically constituted by teachers from outside the country must be organized in such a way that their work leads firmly to the eventual local composition of the group. In any case it is necessary to realize that the contracting of foreign professors represents the only activity for which there is no sufficiently qualified local personnel.

I might say that an ideal solution would lie in equal participation of domestic and foreign professors, the latter being visitors only or contracted in a very permanent way. But this desirable situation probably only occurs after the preliminary difficulties have been overcome. For that reason I shall not discuss it at length. Whether the initial group be comprised of domestic personnel, contracted personnel or both, one of its most important functions will be that part of the general program of putting together the staff of teachers and researchers which we shall call the program of personnel training. By this we mean specifically the series of tasks which the group must carry out for training its own personnel of teachers and researchers. This is the most delicate and important group of functions. In a way the manner in which this function is discharged will determine the true usefulness and importance of the entire effort. If it is properly handled not only will continuity of work and possibilities of growth for the group be assured; they will guarantee the training of new personnel for other activities in the country and particularly for other groups which may be established elsewhere. I should like to look a moment at one of the circumstances surrounding the program of training of teachers and researchers. Basically this program may be thought of in four stages: selection of candidates; training in the local university; advanced studies; organization of the original group.

It is very difficult to formulate rules for the handling of these stages. Nevertheless some comments may be offered. The program of personnel training constitutes a complete task in which the preponderant factors correspond to the stages mentioned:

1. The need to increase the number of candidates and improve selection procedures;

2. The urgent need to strengthen mathematics in schools at the university level to fulfill the requirements of a solid basic education;

3. The need to send outstanding students abroad for the advanced stages of their studies;

4. The difficulties which are often encountered in attempting to induce the graduates to return to their own country.

I shall only deal with them in a general way, emphasizing those which seem most pertinent for immediate development.

As long as we are not able to establish in our countries satisfactory conditions for self-sufficiency it is clear that the education of personnel, at least at the advanced level, will have to be carried out abroad. This brings us to the scholarship program, of which much has already been said. I should like to add that it is desirable to establish a system for the training of groups such as I am describing. Often, while scholarships exist, the conditions they involve are such as to make it difficult to take full advantage of them. It must be recognized that great progress has been made in these programs in the past few years. In particular we should point out the great influence that the advanced centers of more advanced Latin-American countries such as Argentina, Brazil and Mexico have had in the development of science in our countries. I am sure that this influence will be continually more beneficial in the future.

Setting aside questions of sending personnel abroad for improvement, since that has been so amply discussed, I should like to call attention to the problem of selection of candidates and the return of graduate personnel, factors on which the training part of the program rests. We thus reach certain essential aspects which are very closely tied to conditions in the local environment. The implementation of a permanent program of candidate selection for advanced studies assumes close ties between the group and the University in which the group members participate as educators. Only in this way is there any possibility of carrying out the selection in a way that will assure the continuity of the entire program. But this is not the only reason for considering close ties between all the programs and the university as essential. Another not less important reason lies in the aforementioned problem of the return of the graduates.

I must refer to a question that is familiar to all Latin-Americans under the heading of the *exodus* or *emigration of scientists*. This is apparently a foreseeable situation. It has its origins in the great difference in opportunities available in our countries and in more developed countries. And there is a remedy for it, perhaps only one: the establishment of domestic opportunities able to meet the competition with some chance of success. Among the causes for the exodus or rather for the failure of many young men to return is that they have had the chance to improve themselves in advanced environments. We must take note of the attraction of working in an environment that not only offers material advantages but also assures the possibility of scientific activity supported by adequate research organizations in which adequate facilities are guaranteed, as well as the collaboration of other co-workers of real competence.

It is not then a question of mere salary. I think rather, above all, it is a matter of an intellectual order that causes many not to return to the university centers where unfortunately we do not yet provide all the conditions required for development of research.

I believe that the attempt to assure the return of scholarship recipients to the university by means of a legal contract is on the one hand improper and on the other hand destined to fail. I think on the other hand that while one can think in terms of a moral obligation , such an obligation must be balanced by a commitment from the university to establish working conditions such as I have discussed. From the foregoing we may conclude that not only must the entire program be intimately tied to a university but the university should have some plans for creating favorable conditions. Again in this question I cannot enter into detail and I shall merely refer briefly to the series of obligations which the university should permanently accept if it proposes to contribute to the success of the programs I have described: It must constantly advance - as much as possible - in its educational function so that the training of its best students will be the concern at the most advanced levels and in research. And an attempt must be made to eventually focus attention on these levels as well as the others. It must offer teaching and research positions to persons of recognized competence in these areas, offering respectable compensation, stability and opportunities for advancement. Finally, it must afford researchers the indispensable conveniences for their work: time, space, properly equipped library, assistance for visiting professor programs and trips abroad, etc. Even though all that has been said has been in reference to the establishment of activity in mathematics, I believe that at least in part, perhaps with some obvious modifications, these remarks are applicable to other scientific fields as well.

Surely much of what I have expressed reflects conditions that are not operative in other countries and correspond only to the reality in which I have been functioning. My only excuse would be that after having heard the incessant repetition, for many years, of solemn declarations about the need to develop research and having seen this need explicitly proclaimed in our own resolutions, nevertheless very little has yet been accomplished in this direction. I attribute this in large part to the fact that the innumerable and delicate problems of a human, moral and material sort that are posed if one strives to discharge the task well, have not been clarified in sufficient detail.

It can be seen that in fact we are dealing with a long-term labor. But I am firmly convinced, as I am sure many of you have been even before this conference, that the cultural and material development of our countries depends significantly upon the execution of this task and similar ones relating to other scientific and technical fields. There is ever-widening acceptance of the idea that underdevelopment lies largely in the mind and that by exercising sufficient influence upon the minds we may help to overcome the barriers which stubbornly oppose progress.

B. On Mathematics Improvement

STUDIES FOR REFORMS IN MATHEMATICS TEACHING IN SPAIN

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1. Chronology.

Since 1960 week-long annual meetings have been held with intermediate level and University level teachers on the teaching of mathematics at the secondary program. During 1961-62 similar meetings were held for the teachers of elementary-teacher training schools; these were repeated the following year. During 1961-62 a National Commission for the Study of Reforms in Mathematics Instruction at the Intermediate Level was named; the commission is presently pursuing its investigations. This commission developed notes for mathematics courses at the undergraduate fifth and sixth year levels (the first and second year of the preparatory level - student's ages fifteen and sixteen repectively) which were tried out in various public centers. In October 1962, a proposal was agreed upon with O.E.C.D. for the publishing of pilot texts for the first and second undergraduate years (students' ages ten and eleven respectively), and the trying out of these texts in various public and private centers as well as the organizing of teacher-education courses in these cent-education courses of this kind were set up in the University of Madrid (7 courses) Zaragoza (3 courses) Barcelona (3 courses) Navarra (3 courses). Total attendance was 4,8000. These courses lasted from 14 to 28 days and each was devoted to one of the elementary undergraduate courses (4 years, students 10 to 14 years old). It was intended that the O.E.C.D. project should continue through 1965-66, but the aforementioned organization cancelled all its projects related to the improvement of instruction at the intermediate level, interrupting both the work and the publication of the pilot texts for the third and fourth courses of the undergraduate program. These studies revealed that it was necessary to modify the study plan of the graduate degree program in mathematics in the School of Sciences, as will be explain further on.

Although the present study plan at the intermediate level is disorganized, it has not been deemed opportune yet to modify it, since new structuring would require, at the outset, a solution to the problem of adapting the faculty and then carrying out an over-all study of the entire intermediate instructional program to properly coordinate the various subject of each course of study as well as their schedules.

The Objectives of Mathematics Study at the Intermediate Level

The vital question involved in the problem of organizing instruction at the intermediate level is the formulation of the goals sought at **that** stage and in particular in mathematics instruction. One of the common errors incurred in making regulations about course content lies in the failure to attack the problem at its core. Rather, feeling the urgency of remedying a defective situation people pose concrete problems such as the following: printing a new syllabi or programs for some subjects, eliminating some subjects, and subtituting others, (generally with the result of a larger total number of courses) etc. We feel that these methods will never succeed in improving instruction at any level, which is one of the reasons why international conferences such as this one, which attempt to study the technical aspects of instruction, are of great value in orienting governments of various countries with respect to their educational policies. The Commission for the Improvement of Mathematics Instruction at the Intermediate Level in my country, of which I am chairman, has constantly struggled with the problem of clarifying objectives of intermediate level instruction and particularly mathematics instruction at this stage. In the first place, it must be noted that in the intermediate phase, the student is in the midst of a profound evolution in his physical as well as intellectual development; this is the real cause of the difficulty encountered in instruction at this level. Only at the end of this period of learning can the student acquire a sufficiently clear vision of the society in which he lives and of its problems to be able to decide his professional future. This observation may be stated as follows;

A. "At the intermediate stage of instruction the student subject is in a period of maximum physical and intellectual development. Consequently instruction must be so organized as to facilitate this development and produce as complete a man or woman as possible. To accomplish this it must be determined in what proportions we should mix the different disciplines - the intellectual disciplines, the artistic, ethical, social, physical manual disciplines- in order to achieve an harmonious development of the individual's capacity".

We believe that this problem of the harmoniousness of human development is fundamental, since the individual is the object of instruction not its slave. To forget this may be exceedingly harmful, particularly in a meeting of specialists of a given discipline.

The man of today and of the immediate future must possess a totality of basic ideas and knowledge that has been elaborated by humanity to date and has been shown to be useful. Therefore, instruction has an informative dimension inasmuch as it must transmit this store of knowledge to future generations.

The problem that its presents is one of adequate selection of the truly representative ideas of the present state of human knowledge. It is well known today that the way to increase the capacity of memory, human or other, is by organizing knowledge adequately; and this may be accomplished through the human capacity we call abstraction. Yet it must not be forgotten that knowledge is useful only insofar as it may be utilized, which means that it must be observed in action in the solving of the problems that have given rise to it. This leads to an active and ordered presentation of knowledge to achieve economy of memory on the one hand and contribute to the development of intellectual capacity on the other. We may sum this up:

B. "Instruction is an informative dimension inasmuch as it is a vehicle for the transmission of knowledge. The great amount of today's knowledge demands adequate selectivity. In addition, since the transmission of knowledge must be dynamic, it is necessary to present the knowledge in action in the solution of problems; and for economy of memory it is necessary to present it well ordered and with the proper relationships between elements of the same or different disciplines". One of the human capacities that we must develop in students at the intermediate level is the ability to think with clarity and precision. To achieve this, any scientific discipline may be employed, since mathematics being the oldest science and therefore the most worked out, offers two advantages over the others, on the one hand, mathematical structures are simpler and on the other hand, the problems to which they give rise are easier to concretize, which makes it possible to do observation and experimental work with simple and available material. This last reason is important if one bears in mind that in all countries intermediate level instruction involves a very large student body which continues to grow even larger. Consequently effective teaching of Physics, Chemistry, or Biology which require a minimum amount of experimental operator per student at significant expense is costly. As a result, in practice, most institutions give this instruction on a verbal plane which does not meet the minimal conditions for the development of the intellectual capacity of the student.

C. "In the educational aspect of instruction the study of scientific disciplines must contribute to the development of the thinking capacity of the student. This requires that the instruction be dynamic and related to concrete problems which give rise to basic theories. The observation and analysis of these theories must be the source of the appropriate concepts for formulating the laws that make it possible to solve the problems. This process of scientific thinking is seen most clearly and simply in mathematics, which has achieved the simplest structures and standard symbols which facilitate the analysis of problems and the formulation of basic concepts. For this reason, mathematics seems especially apt for providing mental activities which help develop the thinking capacity of the student and give him sure means for achieving clear and ordered thought.

These advantages of mathematics, which are clear to mathematicians, are not as obvious to other specialists, who see mathematics as no more than a calculating process or an interminable chain of syllogisms. Both of these processes correspond to the automatic aspect of mathematics, of little worth from the point of view of the education of the student. It is necessary, therefore, to insist on presenting the true business of mathematics since this is the only way that we can convince others to allow room at the intermediate level for the work of the mathemaitcs discipline. However, it must be noted that there is some real basis for the lack of confidence in the educational value of mathematics, since it has been presented to students as a mummified discipline. We must remember that the Elements of Euclid has been the text book for centuries at the intermediate level and that the geometry that is taught today in nearly all countries, mine included, is a product of that book. The Elements of Euclid is a monumental work, a milestone for humanity; but precisely for that reason and because of the temporal distance between it and our time it is not suitable for young people. Fortunately mathematics has achieved simpler and more accurate forms of expression that are therefore more properly pedagogical. And the most important characteristic of mathematics in our time is its vitality, the consciousness of the mathematical process and the consequent emancipation from rigor mortis and dogmatism. This is the basic reason for the need to substantially modify mathematics instruction. Nevertheless it would be dangerous to base the modification on the growing usefullness of mathematics. The use of utilitarianism as a guide would lead to even worse results than dogmatism and syllogistic automatism.

D. "The fundamental mission of mathematics in intermediate education is a formative one. It should serve to develop a capacity for observ tion, analysis, abstraction, symbolization and the construction of adequate mathematical structures for the study of the problems presented. We feel it desirable that the knowledge communicated to the students form one unit, that its sequence not be worked out independently in each discipline, but rather that it be presented as unified thought and scientific knowledge in all it aspects. For example, if the relationship of equivalence that the student studies in mathematics class cannot be applied in grammar or natural science, etc., the instruction cannot be thought of as being truly organized. The same applies if the mathematics class does not utilize problems from other disciplines as giving rise to mathematics concepts and theories".

It is well known everywhere that instruction in physics faces the problem of traditionally needing the concepts of derivatives and integrals at the beginning of the course, while this concept is taken up at the end of the course in mathematics. Since instruction has been organized from the viewpoint that scientific thought is parceled out in a share-cropping system and that each teacher must not step out of his patch of ground, since he has no business in his neighbors' fields and because his neighbor would protest such an intrusion, the problem has no solution. But it is not only this little problem that has no solution; the total problem of instruction has no solution. Scientific thought, and I think all thought, has one structure only. Either this unified thought is communicated to the student or instruction has not taken place. Can there be a better way of introducing a concept than via the need for solving the problem? If this is the case, what better opportunity could be found for introducing the concept of derivatives and integrals than the physics problem which demands the use of these concepts? It would be unpardonable for the physics teacher not to know how to make the opportune formulization of the concept of derivatives or the concept of integrals through the problem which gives rise to them. The mathematics teacher may or may not use other problems as the points of departure when producing those concepts; and one might even take advantage of the chance to show the student analogies between problems of very different origins as well as the process of mathematical thinking that is followed in each case.

3. Modernization of Instruction And Pilot Text Books.

Once the need to remodel instruction at all levels-and especially the secondary-along the lines of the general premises just discussed was recognized, it became initially necessary to make these ideas known in all countries and attract faculty to them. The problem is not simple. Too many subjective elements are involved for it not to be delicate. But we believe there is no other way. The work of the teacher is by nature very delicate, and the educator may accomplish self-improvement if he is convinced of the worth of the new ideas, but never if those ideas are imposed upon him. Therefore various workshops were organized in the universities willing to cooperate, taking up the problems of instruction in mathematics. In the first workshop, general themes such as proportion, similarity, the measuring of magnitudes, natural numbers, whole numbers, rational numbers, polynomials, irrational expressions, etc. were analyzed.

About 300 public and private school teachers attended. The format for all the courses has been a one-hour lecture followed by 30 minutes of discussion. At the beginning of the course some participants thought they would be given magic formulas for implanting the theorems, formulas and prescriptions required by the official programs in the heads of the students. Others thought that the object was to complicate matters by making them "very abstract", etc. We found it possible generally to make the participants recognize the existence of instructional problems. To convince them, we realized we had to show them. This led us to trying out model courses, starting with the first undergraduate course (age ten-eleven). In view of the age of the students the instruction was obviously experimental. The problem of selection of materials arose not only for the first course but for the entire elementary phase of the program (four courses, ages ten-fourteen). I do not think there is a single solution for the selection of adequate materials for instruction. I believe that many different solutions may prove equally sound and that in the final analysis the worth of the teacher with respect to instruction. Nevertheless, to influence teaching it is necessary to demonstrate with models that have possible solutions and in which concrete problems of how to proceed are solved, examples are given in detail, and concepts are presented. Any other procedure is ineffectual with teachers. And since we believe that the presentation of a model requires prior experimentation, the pilot courses have been written up after being used experimentally in four courses with different teachers.

One of the first problems is the selection of materials for each course. The theme which we feel is basic for all courses is sets, relationships and mappings. This theme is cyclically developed throughout all the undergraduate courses beginning with sets of material things and obtaining basic properties experimentally. The semi-ring of natural numbers is studied in the first course, the ring of whole numbers in the second. The body of rational numbers, as well as first-degree equations is studied in the third course; and the fourth course deals with polynomials, algebraic fractions and second-degree equations. In geometry, the first course is devoted to material construction of elementary geometrical figures and their terminology as well as to the derivation of principal properties of these figures experimentally, using only the relationship of equality. We feel that the construction of basic geometric sizes and the separation of the concept of size from the problem of measurement of size is important. Hence, the second course is devoted to construction of longitudinal magnitudes, angular magnitudes of arcs and polygons. The Greek concept of magnitude is equal to the concept of a determinate class of semi-groups, and we have preferred to adopt the more accurate modern name. The problem of the measurement of elementary geometrical magnitudes has been scheduled for the third course. In the fourth course constructions are made using student sketching, the vector plane; and the basic measurable relationships of the Euclidian plane are obtained by the scalar product of vectors. Fundamental relationships of solids are experimentally developed during the fourth course. Let us analyze each course in detail.

First course (students ages 10-11)

1. Sets Definition of a set. Elements of a set. Belonging relationship. Notation. Venn diagrams. Relations of inclusion. Equality of sets. Disjoint sets. Union. Instruction. Product set. Mappings. Bijective mappings. Relation of equality.

2. Addition and substraction of natural numbers. The cardinal number of a set. Addition of natural numbers. Use of parentheses. Associative property. Commutative property. Graphical representation of the number of a set. Systems of numeration. The decimal system. The metric system. Linear magnitudes. Addition and substraction in a decimal system.

3. Construction, Analysis and Classification of basic plane Geometric Figures. Plane. Straight line. Half-plane.

Point. Construction of a ruler. Half-line. Segment. Chains of consecutive segments. Polygonal line. Angular region. Angle. Consecutive angles. Adjacent and vertical angles. Triangle Consecutive triangles. Quadrilaterals. Polygons. Concave and convex polygons. Circle (disc), Circumference. Area of a circle.

4. Multiplication and Division of Natural Numbers. Multiplication. Properties. Exact division. Practice in multiplication. Integral division. Powers of natural exponents of natural numbers. Decimal numbers. Operations.

5. Transformations in the Plane. Symmetry. Product of symmetries. Motion in the plane. Relations of congruence in the plane. Right angle. Perpendicularity. Construction of a square. Right triangles. The bisection of an angle. Isosceles triangle. Attitudes of a triangle. Medians, perpendicular bisectors, and angle bisectors. Equilateral triangle. Parallelograms. Rectangles. Parallel lines. Square and rhombus. Classification of quadilaterals. Symmetries of the circle. Center, radius, diameter. Compass constructions. Construction of regular polygons. Construction with ruler and compass. Congruence of triangles.

The idea of the plane is introduced by placing a piece of paper on the desk and then adding as many sheets as one may wish to make the desired figures. A straight line is a fold in the paper. The intersection of two straight lines is a point. The student colors the two parts which result from folding the page and thus attains the concept of the semi-plane and the edge of the semi-plane. Through the intersection of semi-planes, angular space and the triangle are formed. Consecutive triangles build polygons. The student cuts out disks of paper and has patterns for circles; the edges are circumferences.[1]

Plane transformations are attained through tracing, for which the student uses transparent paper. Two figures are equal when one may be obtained from the other by tracing. Having the relationship of equality in the plane, the principal relationships between plane figures in which the relationships of equality is involved are obtained experimentally. Conveniently placing two equal triangles in consecutive position, a parallelogram is obtained which serves as a basis for introduction of the idea of parallelism. In this way, the drawing of parallel lines and the use of the square - which the student can himself make from cardboard - arises in a natural way. We believe that at this age the student should be spared definitions that are not constructive. For this reason the construction of parallelograms precedes the definition of

[1]In the U.S.A., one uses circular region or disc for circle, and circle for circumference. The word circumference means the linear measure of the circle.

parallelism. The study of geometry in this course ends with the criteria of equality (congruence) of triangles.

Second Course (students ages 11-12)

1. Sets and relations: Relation of inclusion. Union of sets. Intersection of sets. Product sets. Relations. Equivalence relation. Partition of a set. Mappings.
2. Geometrical magnitudes: Segments. Partitioning into a set of all segments produced by the relation of equality in the plane. General segments. Addition of general segments. Subtraction of general segments. Inequality of general segments. Multiplication of general segments by a natural number. General angles. Addition of general angles. Subtraction of general angles. Inequalities. Multiplication of an angle by a natural number. Polygons. Equivalence of polygons. Partition of a set of polygons with respect to an equivalence relation. General polygons. Addition of general polygons. Subtraction of general polygons.
3. The Integers: Definition. Addition. Use of parentheses. Properties of addition.
4. Isomorphism between semigroups of elementary geometry: Isomorphisms between arcs and angles - angles inscribed in a circle. Isomorphism between the semigroup of general polygons and the semigroup of general segments.
5. Multiplication and division of integers: Definition. Properties. Exact division.

Relations of inequality in elementary geometry: Sum of the angles of a triangle. Exterior angle. Relation of inequality among the sides and angles of a triangle.

Powers of integers: Divisibility.

Initiation into the geometry of space: Rectangular solid. Parallelopiped. Incidence in space. Intersection in space. Parallelism of lines. Parallelism of a line and plane. Parallel planes. Line perpendicular to a plane. Theorem of the three perpendiculars. Dihedral angle.

Thus, as the first course may be considered as being organized around the relationship of equivalence, the second is organized around the relationship of sequence. The concept of magnitude is fundamental in all science and in its Greek sense coincides with a particular type of semi-group. The importance of the concept requires that it be developed carefully in the case of the simplest sciences, which are the geometric ones, since the other scalar magnitudes may be dealt with analogously. It may be said that the geometric part of the course is devoted fundamentally to the construction of these magnitudes. Since the corresponding semigroups are semigroups with difference, difference may be used to define an ordered relationship for every one of the sciences. Through this relationship of order the basic theorems of inequality may be obtained for the plane. The arithmetic section is devoted to whole numbers and to achieving the appropriate automatic manipulation of them by the student. Relationships of incidence, intersection, parallelism, and perpenticularity in space (solids) are introduced by observing straight lines and planes in a paralleliped constructed by each student, if possible of plastic material (clay, for example).

Third course (students, age 12-13)

1. Sets. Inclusion of sets. Implication between propositions. Implication in stochastic events. Union of sets, of propositions, and of stochastic events. Intersection of sets, of propositions, and of stochastic events. Product of sets, relations of order. Segment. Convex figure. Uniform functions. Mappings. Statistical tables. Integral function of integral variable. Differences. The linear function.

2. Proportionality of Segments. Similarity of polygons

3. The Rational Numbers. The fraction as an operator. Equality of fractions. The rational number. Operations with rational numbers. Properties.

4. Measure of Segments. The concept of ratio of two segments. Operation with ratio of segments. Measure of a segment. Approximate measure of a segment using rational numbers.

5. Linear Equations. Linear equation - Systems of linear equations - Applications to commerical arithmetic.

6. Area of Polygons. Area of triangle. Area of a polygon.

The Tetrahedron and Polyhedral angles. The tetrahedron. Trihedral angles. Relation between faces and angles of a trihedral angle. Polyhedral angles. Pyramids. Volume of the tetrahedron and of pyramids.

Fourth course (students, age 13-14)

1. Sets. Boolean algebra of parts of a set, of propositions, and of stochastic events. Frequency of a stochastic event in a sample. Probability of a stochastic event. Expectation (mathematical)

2. The Plane of Free Vectors. Vectors as oriented segments. Relation of equivalence. Free vectors. Addition of free vectors. The group of free vectors in a plane. Multiplication of a free vector by a ratio. The same for a rational number Properties. Linear dependence and independence. Bases of a vector plane. Coordinate of a free vector.

3. The Ring of Polynomials of One Variable Over the Rational Field. Definition of Q [x]. Divisibility in Q [x] . Roots or zeros of a polynomial. Linear factorization of a polynomial. Polynomial functions. The parabolas of second and third order.

4. Cartesian Coordinates of a Point in a Plane. Equation of a line. Linear problems with lines.

5. Equations. Equivalent equations. Solution of an equation. Graphical methods. Abaci (nomograms). Equations of the second degree. Relation between the roots and coefficients.

Trinomial of the second degrees.

6. Scalar Product of Free Vectors. Properties. Modulo of a vector. Angle of two vectors. Orthogonality of vectors. Theorem of Pythagoras. Trigonometric ratios. Fundamental formulas of trigonometry.

7. Powers of Integral and Rational Exponents. Properties. Calculations with radicals.

8. The Concept of Volume. Volume of prisms. Round solids. Volume of round solids. Areas of the surfaces of round solids.

The generating idea of the third course is the measurement of a magnitude. This idea is developed for straight-line segments and by means of isomorophism obtained in the previous course between segments and polygons. It is automatically extended to polygons giving us the concept of the area of the polygon as well as the basic formulas. In all the text books that I know which deal with proportionality of segments and similarities of triangles, the basic point is clearly missed, apparently not being found in the elements of Euclid. This consists of utilizing the measurements of segments to establish proportionality when the measurement is needed for measuring the segments; since we must not forget that at the student level the body of real numbers is not known, so one can not begin by postulating the extistence of a bijection between the points of straight line and real numbers. On the other hand, we consider it completely improper procedure at this level of instruction to introduce real numbers by a system of unjustified axioms. The body of real numbers is sufficiently complicated to require clear reasons for being needed and we believe that the most convincing reason for students lies in showing them that the process of segment measurement may be applied by reason to all scalar magnitudes, but that in that way we obtain a body (or semibody) for measuring each magnitude; which means that the measurement of different magnitudes cannot be related. But since it is a fundamental problem in all science to obtain these relationships, a universal body is needed that will contain all the bodies of measurements of all the scalar magnitudes; and this universal body is the body of real numbers. The rational number is an operating element in the set of whole numbers and as such should be introduced on all levels of instruction including the elementary level (in the case of the elementary instruction we operate solely with natural numbers). The introduction of rational numbers as operators is entirely parallel to the procedure of constructing a semibody of the ratios fo segments, starting from the semigroup of the segments. As the two constructions are carried out in the same course, it is possible to bring home to the student the analogousness of the situations, one in arithmetic and the other in geometry, as well as the parallel nature of the solutions. The formula for the areas of triangles, rectangles, trapezoids, etc. are not introduced until now (student age 13 years). This placement of the most important formulas of all geometry according to the present sequence in my country has given me much to think about. Obviously the weight of tradition is great, but it is not a scientific or pedagogical reason. Naturally, I think that the idea of area should be kept out of the elementary school until age 12-13. As one might assume, I have already heard all the possible objections to this way of proceeding. This is not strange, inasmuch as all the school geometry books and a good part of the undergraduate program lean on what seems to be an important and attractive activity for the student, the calculation of areas of triangular, trapezoidal and other-shaped wheatfields reflecting the imagination of the author of the book. I know of no geometry books for elementary school in which anything is done with the triangle besides calculating its area or its perimeter. It is clear that proposing another procedure must incur many protests. We believe that areas should not be calculated until the problem of the measurement of magnitudes can be taken up with a little care. We advance the following reasons:

1. The concept of area is subordinate to the more general idea of the measurement of magnitudes.

2. The concept of the measurement of magnitudes is a complicated one which, therefore, all precautions notwithstanding, cannot be introduced before the age of 13.

3. The calculation of the area of triangular or pentagonal fields holds no attraction for the student.

4. Very few of the students in schools will ever be surveyors or members of professions in which they will have to calculate the areas or angles of more complicated figures. Obviously none of them will ever have to caluulate them before he is thirteen years old.

5. The metric decimal system is poorly learned because it is customarily presented all at once, which is boring and confusing to the student. We believe that linear units followed by square and then cubic units can be presented gradually and sequentially. It is not difficult to teach 8 or 9 year olds to manipulate square and cubic measurements, but it must be observed that this question of training the student to manipulate relationships among differing units of area is very different from the problem of utilizing the concept of area. There is no reason for the student not becoming familiar and automatic with the relationships between different units of area without calculating the area of triangles, which is merely applying a formula which is not meaningful to him. Linear equations are quickly and

easily studied once the properties of the set of rational numbers have been obtained. The traditional problems of business arithmetic which used to be studied prematurely may be treated as applications of linear equations.

The review of the idea of set seen in previous courses may be enriched by the succinct introduction of operations with propositions and stochastic events. Were I to select only one of these topics I should be inclined to prefer the last one. It permits greater development of the concept of relationships of order leading to definitions of convex figures in an ordered set. While dwelling on the concept of application we can take advantage of the opportunity to bring up the idea of statistical tables and have the student construct some of them following illustrations by the teacher. The manipulation of whole functions with whole variables is a useful exercise. We believe the same to be true of linear functions and their graphic representation.

In the fourth course, Boole's algebras for parts of a set or propostiions and stochastic events may be introduced. It may be used for only one of these topics, depending on the time available in the course. Since measurement of magnitudes has been taken up in the previous course it can be applied as an idea for introducing the concept of probability. By having the student draw on paper, we can introduce the relationship of equivalence between oriented segments of a plane and construct the quotient set. Addition can be graphically defined with free vectors; the same may be done for multiplication by ratios to obtain the eight properties of vector space by student sketching on paper. All of the constructions made by the student give him a clear idea of related planes and a certain skill at outlining. The scalar product and its three properties condense all the metric properties of the plane. The Pythagorean theorem and the cosine of the sum of angles can be made explicit. We do not feel that it is indispensable to introduce other trigonometric functions beyond the cosine. We believe the cosine is enough for the study of all of the mensural properties including geometry at this level; but the sine and tangent may be introduced if one wishes. The study of polynomials is complicated by the problem of an adequate definition at this level; the same holds for the study of equations, especially those of second degree. In the third course, it is observed that the product of the area of the surface of a tetrahedron and the corresponding altitude is constant: this permits the definition of a mapping of the set in the set of ratios which can be seen to have properties analogous to a measurement (the only defined one for tetrahedrons) which we may call volume. The coefficient 1/3 is not justified in the third course. In the fourth course the idea of volume

It should be pointed out to the students that one must prove that the volume does not depend on the subdivision of the polyhedron into tetrahedrons, although that proposition should not be demonstrated.

As was stated at the outset, the initial task of the commission was concerned with the upper undergraduate section (ages 15-17) for which notes were formulated for the fifth and sixth undergraduate courses. But it was found that these courses could not be followed through without prior formulation of those of the early undergraduate phase. The formulation of the later phase, then, had remained unfinished. The formulation we have seen for the elementary undergraduate permits a new structuring of the later phase. The general lines of the notes which were formulated were as follows:

Fifth course. <u>Sets</u>. The natural number. The integer. The rational number. The vector plane. The affine plane. Polynomials. Concept of the real number, operations. Real functions. The power, exponential, logarithmic, and circular functions.

Sixth course. <u>Sets</u>. Probability. Limit of the function of a real variable. Continuity. Derivatives. Differentials. Increasing and decreasing functions. Maximum or minimum. Scalar product of vectors. Metrical properties of the plane. Trigonometry. The conics. Definite integral.

Theory of integration. Applications.

4. Undergraduate Specialization.

Possible specialization at the undergraduate level has been much debated. The present situation in Spain provides some variation in arts and sciences. The elementary undergraduate (10-14 years old) is common for all. The upper undergraduate (15-17) has the specializations we mentioned even though their final degrees may not bear the same terminology. That is, a student with an AB in liberal arts can enroll in the school of sciences and vice versa. With respect to the mathematics, there is no mathematics subject in the liberal arts undergraduate program. From which it can be gathered that we do not favor specialization at the undergraduate (preparatory) level. We believe that specialization can be justified with the following reasons:

a) The difficulty of achieving an adequate organization of subjects compatible with the class schedule and study program suited to the physical and psychological characteristics of the student.

b) The utilitarian point of view. To orient the student as early as possible to his future professional activity, avoiding wasting his time with study that he will not use in the future.

c) Catering to the specific capacities of the student or his possible future vocation by placing him in a study enviroment related to his natural predispositions and therefore more likely to produce results. Undoubtedly each one of these viewpoints can be defended, but I believe they should be more carefully analyzed to discover possible drawbacks.

d) The enormous quantity of present knowledge of positive value for humanity makes an adequate selection truly difficult.

This difficulty becomes insurmountable when the mission of instruction is conceived as informative. It would certainly be marvelous to produce an undergraduate with clear, strong [illegible] the cosmos, living organisms, diseases, hygiene, [illegible], the history of civilizations, the organization [illegible] law, the evolution of scientific thought, art- [illegible]stations, music, painting, sculpture, architecture, [illegible] psychology, etc., but obviously this is not [illegible] and an attempt to achieve it wauld lead to a sterile [illegible] of instruction. If to avoid this difficulty, [illegible]ions are introduced one would have to provide as [illegible]ons as there are professional specializations, [illegible] unfeasible, bearing in mind the great number [illegible] involved at the intermediate level compared to [illegible]en in by the upper level institutions. One [illegible]hat the question has been posed in an extreme form and that no one thinks in terms of such a solution, that the idea of specialization is related to the present content of curricula and that all that is suggested is an adjustment of the over-extended study plans to pedagogical reality. But even if that be so, we do not see a satisfactory solution possible from the informational point of view. For me the following reasons invalidate the informational position with respect to the problem of instruction at the intermediate level:

1) It is very difficult to decide which elements of present knowledge have greater informational value;

2) Accepting such a decision as possible, the modern state of evolution of scientific and literary thought would require frequent changes of subject and techniques, which would constitute an operational difficulty for institutions and presupposes very special conditions for the facutly, which can hardly be assumed when we are dealing with so large a group of teachers as intermediate instruction requires;

3) The information acquired at age seventeen will be useless assuming it endures in memory--at age 40;

b) The utilitarian point of view encounters the following difficulties:

1) It is practically impossible to organize an instructional program with a sufficient number of specializations

to future professions.

2) The orienting of instruction toward professionalization would lead to a situation analogous to the art guilds of the Middle Ages (which produced very little cultural fruit) and would shut off possibilities for scientific and technical progress.

3) It is very difficult to decide the future profession of a student of age 14 or 15;

c) The specific ability of the student in the undergraduate stage is often conditioned by the skill of the teacher, he has had. It is quite common for a student to think in terms of a literary vocation because of the influence of a good teacher of literature in his school; what is really going on is a reflection of the enthusiasm of the teacher. It is possible that the differences between the mental activities involved in different disciplines are much smaller that what is assumed; this would seem to be indicated by the fact that many students show excellence in literary and scientific activities at the undergraduate level--in which case there would be no reason for this defense of specialization.

Nonetheless, if instruction at the intermediate stage s centered around the aspect of intellectual formation; if t is thought desirable to achieve, as a result of this, a an or woman able to think with clarity and precision, to express himself correctly and clearly in his own language; who will have achieved a healthy physical, psychological and moral development which will make it easier for him to adjust and collaborate within society; if this is to produce men and women able to recognize their mistakes and find remedies, with a developed will power to achieve and to fulfill tasks notwithstanding the difficulties that arise; if this is the aim, then the problem of instruction at all levels does have a solution. The solution is neither easy nor definitive but it does open the way for constant and positive advance.

5) The problem of faculty. As has been previously stated, informational courses for teachers at the intermediate level were organized. These courses were successful in making an important segment of the faculty, public and private, at the intermediate level, aware of the need to bear in mind the objectives and methods of mathematics instruction at their intermediate level. The nature of these courses could not be better. By themselves, they cannot transmit the knowledge or the techniques to advance the evolution of instruction. This would be possible only if the faculty had a background of modern mathematics, which is not the present case. For that reason I brought up the need to organize content courses lasting two years to bring the faculty up to date. The idea was well received by a significant portion of the faculty, but I found little support from ed-

set up such courses as yet. Since this emergency plan failed we addressed ourselves to longer-term solutions. The official degree required for joining the faculty of an intermediate instruction institution, whether public or private, in Spain is the Licentiate in arts and sciences with a major in one or another discipline. This degree may be considered as more or less the equivalent of the masters of science in the United States. There are five different science degrees: Mathematical sciences, physical sciences, chemical sciences, geological sciences, and biological sciences. All of them carry the same weight, which means that a teacher with a degree in biological sciences is authorized to teach mathematics and vice versa. The licentiate in mathematical sciences, except for some possible small variations, was special and was thought of as preparing a pure mathematician rather than an intermediate level teacher. The premise in that was: "He who knows most knows least", which may be true but which in practice resulted in the mathematical sciences study program being much harder than those of the other degrees. From the public school point of view it was the equivalent of the secondary instructional level. One of he results was that the majority of the teachers of mathematics n private institutions (which include 80 per cent of the tudents of the country) were graduates in chemical and bioogical sciences. If we bear in mind the fact that graduates in chemistry have the same first year plus some supplementary mathematics in the second year, it can be seen that these specialists will be inadequately prepared in mathematics for teaching this discipline in institutions of the secondary level. In addition, the study plans that we had did not effectively take up pedagogical problems and did not include elementary theory of divisibiltiy, theory of proportionality of segments, concept of the area of polygons or the volume of polyhedrons, etc., items which must be included in any syllabus of mathematics at the secondary level.

To meet these needs, the study plan was modified in 1964 in the science mathematics licentiate at the university of Madrid, establishing three majors: Pure mathematics, applied mathematics, and methodology and pedagogy of mathematics. The three specialties more or less share the same first three courses and differ in the last two (the licentiate requires five years). The following is the study plan for the licentiate in mathematical sciences with a specialty in methodology and pedagogy:

First year. Linear algebra. Infinitesimal analysis. Physics, Chemistry. Drawing.

Second year. Mathematical Analysis I. (Theory of functions of a real variable; Geometry I. (Vector space, affine space, Euclidean space; projective space, curves and surfaces of Euclidean space), Algebra and Topology (Ideas of Mathematical Logic, Theory of sets, Groupoids, semi groups, Groups, rings, fields, vector spaces, topological spaces, Compact space, Metric space, Banach space, Hilbert space, Euclidean space). Physics I.

Third year. Mathematical Analysis II. (Differential Equations) Geometry II. (Moduli, tensor product, exterior product, bilinear forms, quadratic forms, Applications of Geometry). Calculus of Probability and Mathematical Statistics. Methodology and Didactics I.

Fourth year. Mathematical Analysis III (Functions of a complex variable). Elementary Mathematics I. (Axiomatic construction of affine space and Euclidean space. Study of curves and surfaces in euclidean space; linear programming). Algebra and Topology II (Applications of the type instructed by the ideas in the first course algebra and topology. Elementary study of algebraic topology). Methodology and Didactics II. (Problems of instruction and practice in teaching at a center of intermediate instruction). Statistics applied to education.

Fifth year. Mathematical Analysis IV. (Functional Analysis). Elementary Mathematics II (Theory of measure, [illegible] of representation, Programming of electronic computers). Methodology and Didactics III. Practice Teaching.

ADVANCES IN MATHEMATICS IN CHILE

Cesar Abuauad, Santiago

The Ministry of Education can surely say with complete justice that the need for reform in the mathematics program or the need to improve secondary level teachers has long been the customary labor of the Ministry. But both these matters, reform and improvement, used to have a completely different meaning from what they are beginning to have now. I can mention four points which particularly contributed to this awakening: 1. Wide distributions, throughout the country, of the agreements and deliberations of the Bogota Conference of December 1961; 2. Personal contact of Professor Stone in January of 1962 with the principal educational authorities of Chile; 3. The work of the American mathematical group SMSG whose work has penetrated many sectors in Chilean mathematics education; 4. The spirit of reform from Europe, with which fortunately we still maintain close contact.

The following advances since 1962 reveal a strong, informed movement in mathematical education circles and even in basic sciences at the secondary level:

a) In 1962 the "new mathematics program for high school education" was published. This program is neither new nor of any value, but it is worth mentioning because of the rapidity with which it was officially approved and implemented in a society in which the period of drafts and reports usually lasts

b) Union initiative resulted in the setting up of mathematics seminars in 1963 and 1964 with weekly meetings and the attendance of some university teachers.

c) The State Technical University held two summer institutes in 1965 and 1966 for its own faculty but with some little attendance by secondary teachers. Professor Burton Jones gave two lectures in the institute in 1966. To the credit of its organizers I should like to state that both institutes were beautifully organized and successful.

d) With the cooperation of the Ford Foundation a seminar on the teaching of the basic sciences was held for one week in 1965 at the Technical university Federico Santa Maria. The mathematics Committee approved the general lines of the recent international conferences on mathematics education and outlined a conservative mathematics program which ten years ago would have scandalized the ministry.

e) In October 1964, the national program for improvement of high school and elementary school teachers was established in the Office of the Superintendent of Education. The State considers that the PNP is an essential part of the education of high school and elementary teachers and not merely a supplementary or irregular activity. The superintendent is to propose to the ministry of education, before the first of August of every year, the plan of activities for the PNP (National Program for Perfection of Teachers) for the following year; and the ministry of education is to pass on the proposal before the first of November of the

The executive technical council directs the PNP. It is composed of 22 persons, 8 of whom represent the ministry of education including the Ministry and the superintendent; 5 are from the university of Chile; 6 from the other universities of the country; 1 represents the council of rectors and 2 represent the unions. This council, which supposedly provides the technical orientation for the improvement program, in actuallity has functioned as a censoring and watchdog organism, fearful lest technology go beyond their own convictions. The PNP also has a coordinator and executive secretary and six specialized committees, of which one is for mathematics and five for education The mathematics committee is in charge of the programming of courses, the selection of teachers and assistants who are to give them and the production of written materials as well as even the selection of teacher training personnel.

To provide an approximate idea of the future benefits from the PNP it is appropriate to point out that an improvement center 8,600 square meters in size is being constructed on a campus of 22 hectares and will have four hundred educators from high school and elementary levels.

Technical orientation of PNP	Executive Technical Council 8 Ministry of Education 6 University of Chile 6 other universities 2 unions
Administrators of PNP	1 coordinator 1 executive secretary

Specialized Committees Educational Committees

Mathematics committee

Natural sciences committee

Human sciences committee

Language committee

Technical artistic committee

In January 1966, when the mathematics committee was concerned with the formulation of programs for the first improvement courses that were to be given in the middle of January, it was surprised to receive from the administrators a plan of a "Program of Transition for the Seventh Grade" put together somewhere within the technical Office of the Superintendent's office without the knowledge of the committee. The plan sought to organize a system of courses to cover the transition program. These were designed for 117 seventh grade educators of which 110 were elementary level teachers with no university degrees.

While declaring that this was an unforeseen situation for which the committee was not prepared, the committee made the following observations to the administrators of the program:

a) It was inconvenient to program improvement courses for a given grade;

b) It was desirable not to proceed along those lines;

c) It was desirable to have personal contact with the technical office;

d) Direct participation was requested in the elaboration of the future mathematics programs for the high school, which is one of the interests of some mebers of the committee.

This petition from the mathematics committee, which seemed quite natural in view of the nature of the work with which it had been charged, was repeated a number of times to the administrative authorities; although the coordinator and executive secretary shared the same concern and felt that the petition was a corollary to their own work and education, an article, passed by the Office of the Superintendent of Education in 1954, authorizes that office to do what it pleases in the area of high school and elementary level instruction programs. And although it also authorizes the superintendent's office to consult persons or institutions, national or international, to accomplish its specific work better, the active participation of the committee in the elaboration of a future mathematics program for high school education cannot be assured rapid consent from the superintendent's office. A very intense labor of persuasion awaits the committee.

In organizing the system of courses and methodological laboratories to cover the transition program of the seven grades, the committee had the assistance of Dr. Vogeli, kindly sent by Professor Fehr. These courses, which were continually postponed until the middle of March, were brief; their programs had to be readjusted while in progress; and they provided a magnificent experience for the directors and administrators of the plan. Except for three or four participants, none of the students were willing to sign the only written test given at the end of the courses asserting that if they did sign, poor work might be

The program for improvement of mathematics teachers at the secondary level went into operation the beginning of July of this year with the participation of 37 high school teachers selected from a group of 70 candidates. The primary offerings were two courses of 30 class hours each, algebra and geometry. The first covered the basic structures of algebra up to the concept of vectorial space and linear applications. The geometry course was basically the system of Hilbert's axioms. The July session had a normal development including two written tests and was acclaimed as a great success by the participants and by the committee. All of this has had to operate initially within the narrow framework of material conditions. When the function of the PNP attains the heights hoped for by the Ministry of Education and the reformed mathematics programs for the preparatory school can then be implemented in a natural and spontaneous form and without the hidden or open resistance now encountered in some sectors, we shall have passed beyond that long and dark stage in which Chilean mathematics education at the secondary level has been kept.

I should like to conclude with a few words about mathematics education at the university level, since the spirit of this afternoons' agenda does permit it.

In the last 8 years there has been a considerable surge in university level mathematics education. In the Faculty of Sciences the Department of Mathematics maintains a level of studies of excellent qulity which you can verify in the display of materiwhich normally accompanies these conferences. This faculty, which includes Professor Biberstein and J. Balderrama, currently in Venezuela and graduate of Belgium studies, grants a graduate degree in five years. It makes such rigorous demands that it is the opinion of a noted American mathematician who visited Chile a few months ago as part of a scientific mission, that they are more demanding than the master's degree of many universities.

It should also be mentioned that the Faculty of Mathematical and Physical Sciences under its recently appointed dean D'Etigny has had a great expansion, reaching 600 students in the first year of study and being therefore able to absorb a great number of graduate students in mathematics for its teaching needs. Besides enriching its mathematics programs in quality and variety, it has established a degree program in mathematical engineering, apparently following the French model.

Finally, I wish to mention the State Technical University, which in recent years has made great efforts to strenghten its mathematics programs in variety and content. The universities educational council A of the Department of Basic Sciences will shortly receive a project proposal to create the M.A. degree with a major in mathematics, the work of Professor Mesa, president of educational council A and Dr. Michelow.

PROGRESS OF MATHEMATICS INSTRUCTION IN BRAZIL

Oswaldo Sangiorgi, Sao Paulo, Brazil

Introduction

Since the first Inter-American Conference on Mathematics Education, held in Bogota in 1961 and in the expression of Dr. Jaime Posada in his opening Discourse, A Gathering of American Scientific Thought, there have been a series of good and even magnificent results in the area of mathematics instruction in a great number of countries of the Western Hemisphere.

The training of secondary level teachers in modern mathematics content;

The reformulation of mathematics programs for the high school and corresponding articulation with elementary and higher level instruction;

The elaboration of textbooks with a new approach to mathematical subjects;

have been the goals most intensely sought by those responsible for mathematical education of American youth.

Mathematics Instruction in the Brazilian Secondary System

It is a source of pride to us to be able to consider Brazil among the countries which have emerged from a discouraging picture in these last few years. Broadly speaking, today we boast a progressive phase with

While recalling the famous phrase of Professor Omar Catunda, one of the representatives of my country at the Bogota conference when he opposed the "Down with Euclid" (supported by the illustrous professor Dieudonne and staunchly defended by participants in that congress) proposed for the Brazil of that time "At least Euclid," - we shall evaluate the healthy reaction shown by my country in the years that followed and the consequences.

As the distinguished delegates know, Brazil is more than a country; by virtue of its size it is a continent in this hemisphere. Given this condition, it is not easy to keep exact track of all the progress that has taken place in the mathematics instruction in the 22 provinces, four territories and federal districts of the country.

While some provinces, principally those of the coastal belt (North-Central-South) show considerable progress, the same cannot be said for all the central provinces, with the brilliant exception of the Federal District, Brazilia. Therefore, any statistical table which included overall figures would run the risk of not reflecting the rate of real progress which has been made and which we have been invited to discuss.

One of the major factors directly responsible for the change in mathematics instruction in my country -- and I repeat that the change is sometimes radical in some provinces -- is the new climate found among the universities, the mathematics institutes, the study groups and the public education authorities (Ministry of Education and Culture through the Directory of Secondary Instruction for the Federal Region and the Offices of the Secretary of Education for the provinces) which have made it possible to give greater unity to the reform efforts demanded by the mathematics teachers of the country.

One of the possibilities realized by Brazil was the increasing of the cooperative relationship between the university mathematicians and secondary-level teachers. Study groups were formed that were linked with the universities and sought to improve and modernize mathematics instruction, principally at the high school level.

The first study group of mathematics instruction -- GEEM -- of San Pablo had already been founded (10-31-61) and had begun to attack the problem by: courses for teacher improvement at the secondary level and later the elementary level; the elaboration of a modern mathematics curriculum which could serve as a basis and guide for secondary instruction; editing and publishing textbooks which describe the results of experimental classes in mathematics and other work mentioned in the report on "Improvement of Mathematics Teaching in Brazil" presented by the Brazilian professor Dr. Afred Perera Gomez at the Inter-American meeting in Rio de Janeiro sponsored by the National Science Foundation, November 30 - December 2, 1964.

We may now summarize in numbers, so that everyone may follow more clearly the evolution that has taken place in Brazil:

	1962	1965
Schools of Philosophy, Arts, and Sciences with Mathematics Departments	13	46
Institutes of Mathematics	1	13
Training Centers for Science Teachers and Study Groups on Mathematics Instruction	2	8
Secondary-level Teachers who participated in improvement courses in mathematics	578	7,250
Percentage rate of mathematics teachers at the secondary level who have had higher education	22%	47%

For the province of San Pablo where we have the most accurate information, the data is based on a total of 1,324 institutions of secondary level instruction, of which 686 are public and 638 private. In San Pablo during 1966, we had in service 6,276 mathematics teachers of whom 63 percent held university degrees.

Whereas in 1962, in the province of San Pablo the schools of philosophy, arts and sciences which had mathematics departments were 6 in number, granting a total of 128 graduate degrees in mathematics and physics, in 1966 we had 12 schools of philosophy, arts and sciences with mathematics departments which in 1965 granted 265 graduate degrees in mathematics and physics.

In other words, the number of graduate degrees in mathematics in the province of San Pablo has practically doubled since 1962. Still, the 265 graduate degrees in mathematics and physics do not begin to meet the demand for new teachers in mathematics required by the San Pablo High Schools, especially if we discount those who stay on in the graduate schools as instructors.

In this year alone the secondary schools need almost 400 new teachers.

Therefore, the shortage of mathematics teachers continues and since, with slight differences, this picture is the same for the other provinces, it means that the country is still in need of training increasing numbers of teachers who will be able to handle the new classes, though without the desired higher educati

To see any progress in this very curious matter of filling in the holes in the system of secondary instruction with teachers without graduate degrees we must take not of the careful preparation given during the last three years by the Campaign for the Improvement and Extension of Secondary Instruction (CADES) of the Ministry of Education and Culture to those who intend to teach mathematics in the high schools. In a great number of provinces courses on introduction to modern mathematics are given (with set theory, mathematical logic, and modern practice) as well as improvement courses in mathematics (with modern algebra, applied mathematics, deductive techniques).

Such courses are sometimes directed by the mathematics institutes (as in Paraiba and Bahia), other times by the training center for science teachers (currently located in Recife-Pe, Salvador-Ba, Belo Horizonte-MG, Rio de Janeiro-GB, Sao Paulo-SP and Porto Alegre-RS); still other times they are handled by the study groups attached to the universities (as in the case of GEEM of San Pablo). These courses have particular features which

Principal Component of Progress in Mathematics Teaching in Brazil: Greater Participation of Mathematicians

Professor Luis A. Santalo of Argentina said at the Congress of Bogota (and I quote): "What a good mathematics teacher needs most is above all to know mathematics, and the more the better."

That is why it is necessary that mathematicians from the universities take part in all the scheduled courses to give the future secondary level teacher a live contact with present mathematics. This was a formula we employed.

Meanwhile the Brazilian mathematicians have been fed by institutes for mathematics research (13 in the country today). Outstanding among them is the IMPA (Institute of Pure and Applied Mathematics) located in Guanabara and the first to be created in 1952 by the National Council of Research. The IMPA is currently directed by Professor Lindolpho de Carvalho Dias, with the Brazilian mathematician Dr. Leopoldo Nachbin as chief of research. Through study scholarships, the IMPA has made it possible for a great number of scholars to enroll in masters and doctors programs in mathematics under the guidance of Brazilian and foreign teachers. This is the largest research center in mathematics in the entire country.

Their specific activities as well as those of other institutes and departments of mathematics linked to the universities are the subject of the report on mathematics education in the universities of Brazil, presented to this congress by Professor Leopoldo Nachbin.

The Brazilian Colloquia on Mathematics, sponsored by the IMPA every two years since 1957, has been the center of communications and results for mathematics researchers in Brazil. It has also been the place for advanced mathematics courses given by Brazilian and foreign teachers who are specially invited. The high level of the aforementioned colloquia can be seen in their records which are available from the publications section of this conference for all who are interested.

It is important to point out that the colloquia of 1961 and of 1963 had study sections for mathematics instruction in the high school and its articulation with higher instruction.

In these sections we clarify exactly what is meant by mathematics in order to advise the study groups, which orient the secondary level teachers.

Other meetings which are responsible for the closer relationship between Brazilian university mathematicians and secondary mathematics teachers are the Brazilian Congresses of Mathematics Instruction.

These congresses, begun in 1955 in the city of El Salvador (Bahia), have been a springboard of progress for mathematics instruction in the secondary schools.

To summarize briefly, at the fourth congress, held in 1962 in Belem (province of Para), the GEEM of San Pablo exhibited for the first time in Brazil the initial results of the application of the so-called modern mathematics in the high school, in at that time opinions in Brazil differed as to the applicability of modern mathematics at the secondary level, there is no doubt that that congress was a great stimulus for emulation by some teachers who were equipped for reform but did not undertake it out of timidity.

The fifth congress sponsored by the GEEM in the aeronautics ITA technical center in the city of San Jose de los Campos (province of San Pablo), January 10-15, 1966 was attended by 350 participants from all over the country. For the first time it featured collaboration of noted professors, mathematicians and "experts" involved in the problem of mathematics instruction. Among these we may cite: Marshall Stone (USA), George Papy (Belgium), Hector Mercklen (Uruguay and representing the Inter-American Program for Improving Science Instruction -- PIMEC) and Helmuth Renato Volker (representing the Ministry of Education of Argentina).

The fifth congress included a very objective affirmation of the progress made by a good number of Brazilian provinces in the last three years in the field of modernization of mathematics in the secondary schools. It was also the scene of excellent methodological suggestions brought to the body by noted foreign and Brazilian teachers. Its principal theme: "Modern Mathematics in the Secondary School" permitted the presentation of many papers (some considered daring) as well as demonstration classes with student participation.

In the publications section of this conference there is a report with more details for those who are interested.

Immediate Consequences of the Fifth Congress:

1. The Ministry of Education and Culture, through the CADES, responded to the requests from various provinces and sponsored improvement courses in mathematics. This year (month of July) through the training center for science teachers, 25 improvement courses in mathematics were given around the country for secondary teachers. These so-called 120-hour courses were attended by about 600 teachers.

2. Through agreement with the schools of philosophy, arts and sciences of Ceara, Rio Grande del Norte, Paraiba, Pernambuco, Bagia, Alagoas, Espiritu Santo, Belo Horizonte, Londrina (Province of Parana), Presidente Prudente (Province of San Pablo), Floriano-polis (Province of Santa Catarina), Porto Alegre and Santa Maria (Province of Rio Grande do Sul), CADES has been giving 120-day courses meeting for five hours of work per day and involving general culture and pedagogy in addition to modern mathematics content.

Why culture and pedagogy? Because it is our feeling - to add to the statements of Mr. Santalo -- that besides knowing mathematics the teacher must be in complete command of the language of his country!

Such courses also fulfill the goal of providing candidates for secondary teaching positions and permanent registration to meet the deficit of secondary level mathematics teachers every year.

3. The GEEM of San Pablo, in 1966, in cooperation with the universities, mathematics institutes and class associations, held courses for improvement in mathematics (first, second and third levels) in: Jao Pessoa (Paraiba), Brasilia (Federal District), Victoria (Espiritu Santo), Sao Paulo (Capital), Porto Alegre (Rio Grande do Sul), Franca (Sao Paulo) and Sao Carlos (Sao Paulo). These courses, which involved about 800 level teachers, dealt with set theory, mathematical logic, modern algebra, linear programming, topics of topology, probability and statistics.

Mathematics Programs presently operating in Brazilian Secondary Schools

At present, nearly 70 per cent of Brazilian secondary institutions, including 40 percent established in 1963, offer the following mathematics program:

Ages 11-12 years: Ideas of sets and relations. Operations in the set of whole numbers. Structural properties. Intuitive geometry; easy constructions and problems on bisection of figures.

Ages 13-14 years: Operations on the set R; algebraic calculation; functions; linear function; quadratic function; plane geometric figures; simple closed curves; convex polygon; circle.

(Note: The treatment given up to this point is axiomatic, but in a beginning phase, in contrast to the excessive rigidity which prevailed for many years in the Brazilian secondary school and had disasterous results.)

Ages 15-17 years: Functions; exponential, logarithmic, and trigonometric functions; progressions; combinatorial analysis; matrices and determinants; systems of linear equations. Operations with complex numbers; Introduction to infinitesimal calculus. Algebraic polynomials and equations; analytic study of the line, circle, and conics. Geometry of space; axiomatic treatment.

Nevertheless, there are experimental classes in some provinces offering more advanced programs under the supervision of professors attached to study groups. Such classes exist in Salvador, Belo-Horizonte, Niteroi, Guanabara, San Pablo, Curitiba and Porto Alegre.

As an example, we shall cite what is being done for the last four years in the capital of San Pablo by some institutions of the public system, the State Vocational School, for example, and some private institutions such as the Santa Cruz School, under the guidance of professors of GEEM:

Ages 11-12 years:
1. Relations of order, of equivalence; partitioning.
2. Operations as relations. Inverse operations and the set of integers; structural properties; group structure.
3. Rational numbers, operations, structural properties.
4. Universal set, determining the truth set of numerical sentences; equations of the first degree

5. Measurement; associating numbers to geometrical figures.
6. Affine function; $y = ax + b$, $a \neq 0$, geometric construction.
7. Systems of linear equations of two variables, the use of connectives <u>and</u> and <u>or</u>.

Ages 13-14 years:
1. Real numbers; operations; structural properties; group structure.
2. Set of polynomials; operations; structural properties; ring structure.
3. Quadratic function; $y = ax^2 + bx + c$, $a \neq 0$. Equations of the second degree.
4. Concepts of logic; propositional calculus; preparation for axiomatics.
5. Congruence of geometric figures.
6. Geometric transformations of the plane;
   a) which conserve congruence (translation, rotation, reflection),
   b) which conserve proportion (similarity, homothety).

Ages 15-17 years:
1. Functions; sequences; exponential, logarithmic and trigonometric functions;
2. Induction; Counting problems; probability;
3. Matrices; operations; structural properties;
4. Systems of linear equations.
5. Polynomials (squences quasi-null)
6. Introduction to Calculus, algebraic equations.

In November 1964, the National Association of Mathematics Teachers and Researchers (ANPPM-RJ) of Niteroi, held the Janeiran Mathematics Study and Teaching Week which included the presentation of an experimental paper on Modern Treatment of Geometry in the Secondary School which had been used in local classes. The experiment took place in the educational center in Niteroi with quite positive results.

In Salvador (Bahia), for the second time now, experimentation is going on, including the public schools, with a modern program of the first cycle for secondary schools. This is of an experimental character under the guidance of professors of the mathematics, physics institute of Bahia. The program is described in the report of our delegation. In 1965, the Municipal School of Belo Horizonte (Minas Gerais) started a minimal program of modern mathematics in the first cycle for secondary schools, and this year it is continuing the program with the first and second cycle.

In Porto Alegre (Rio Grande do Sul) the public school "Julio de Castilhos" has experimentally established various areas of secondary mathematics under the guidance of the mathematics study group attached to the school.

In Curitiba (Parana) the local GEEM is coordinating the experiments in the State School (the largest in the country, involving around 6,000 students!) with the first three courses for secondary schools.

Reformulation of Mathematics Instruction in the Elementary Schools

In my country, and I believe in other American countries as well, there is a great problem of extending the new reformulation of mathematics to the elementary school.

In view of the very large body of elementary school teachers (in Sao Pablo alone there are more than 60,000) the solutions are being studied for both the long and short term. Beginning in 1962, twelve courses of introduction to modern mathematics for elementary school teachers were given in various Brazilian provinces.

The elementary instruction section of the GEEM of Sao Pablo has already given courses for more than 1,500 in-service elementary school teachers; this has been extended to include provinces of Rio de Janeiro, Parana and Rio Grande do Sul.

The Secretary of Education of Sao Pablo had been maintaining educational television programs since 1962 at the elementary instructional level. They are on the air daily from 9 to 11 in the morning and from 5 to 5:30 in the evening; and all of them include classes in modern mathematics in sections for teachers and for students. As a matter of fact, the sample of a televised class that we are sending to Japan on the occasion of the "Japan Award" which will be given this year in that country, is on modern mathematics aimed at students in the elementary school. The program was formulated by the elementary instruction section of GEEM in San Pablo.

The solution stated by the majority of the provinces is to send teams of accredited teachers to the interior to train elementary school teachers in the reformulation of mathematics to be taught in the modern elementary school by means of week-long courses with the teachers. This kind of thing is being done also by the cultural expansion service of the education department of San Pablo which gives monthly study sessions on modern mathematics for elementary level teachers from the interior which, jointly with the elementary instruction section of GEEM, is preparing a new mathematics program along modern lines to be begun in 1967 by all the public elementary schools in San Pablo.

In the long run, such a reformulation begins with the preparation of future elementary teachers in the normal schools which exist for this purpose and abound in the country. Every conceivalbe sort of publication related to the modernization of mathematics instruction in the elementary school exists. Even some books on pedagogy of modern mathematics intended for the students have been in use for almost two years, and with good results. Samples of such books are in the publications section of this conference.

Modernized Publications

a) Of an informational character --

In the higher education sector, IMPA, other mathematics institutes, the Mathematical Society of San Pablo, and the Paranaean Mathematics Society maintain regular publications (some of them financed by the National Research Council) which publish work in high level [illegible]

The last few bulletins of the Paranaean Mathematics Society features papers on modern mathematics in secondary instruction.

In the secondary level sector, there are many experimental publications in almost all the Brazilian provinces. GEEM of San Pablo maintains an information bulletin on the principal happenings in the mathematics instruction sector. In addition to this bulletin, GEEM has published:

Teachers Series:

1. Modern Mathematics for Secondary Level Instruction
2. A Modern Program of Mathematics for Secondary Instruction (translation of the O.E.C.E. book)
3. Elements of set theory
4. Mathematical Logic for the High School Course
5. Combinations and Probability

Elementary Instruction Series:

1. An Introduction of modern mathematics in the elementary school

b) Of a pedagogical nature--

The current pedagogical mathematics books intended for Brazilian secondary education, first phase, now deal with mathematics instruction in the modern way, gradually introducing some basic concepts and placing technical operations in their proper place.

Some authors now make a teacher's guidebook with their texts to clarify and scientifically solidify the new ideas as well as modernize them with new methodological instructions.

All these publications are found in the publications section of this conference.

Some Significant Facts

Under the heading "Progress in Mathematics Instruction in Brazil" we feel that some things that have occurred in mathematics education in the last few years in my country are worthy of mention. In Brazil we have bodies like the Brazilian Society of Progress in Science (SBPC) which has a highly scientific tradition. It sponsors annual meetings in various parts of the country involving hundreds of university professors in some way specializing in the sciences. During the last three years, SBPC has programmed sessions on modern mathematics in its meetings during a week in July. They have had repercussions throughout the country. Such sessions were held in Curitiba (1964) in Belo Horizonte (1965) and this year in Blunenau (Province of Santa Catarina).

Similarly, the Brazilian Institute of Education, Science and Culture -- IBECC-UNESCO, San Pablo Section, the oldest science center attached to secondary schools in this country, has a mathematics section which organizes activities for secondary school teachers through courses and publications including Portuguese translations and adaptations by teachers in that sector of the textbooks of the School Mathematics Study Group (SMSG), Mathematics for High School series, Volumes I, II, and III.

An important note: the recent contests of the series "Scientists of Tomorrow" organized by IBECC receives original papers on physics, chemistry, natural history and mathematics from hundreds of students from all parts of Brazil, to be judged by university professors in the annual meeting of the SBPC. The first prizes were awarded in recent years to works on modern mathematics by students of secondary institutions in San Pablo (Capital) and Santos, breaking a long-standing tradition whereby mathematics had never won a prize.

Also in the science exposition which in the last two years has excited our young people, the modern mathematics booths have been the center of a great deal of attraction for the general public.

In January 1964, in cooperation with educational television of the Office of the Secretary of Education in San Pablo, GEEM gave the first televised course in modern mathematics intended for high school teachers. The course was given for two weeks by university professors, with study sessions on set theory, mathematical logic and modern practice. The evaluation tests given by television have been administered in public places under the supervision of school officials, with surprising results.

During the months of September and October of this year, the first course of modernization in modern mathematics for parents was given in the capital of San Pablo by the State Vocational School (of the public system) in collaboration with GEEM.

about 200 parents actively participated in the study sessions. Conference delegates may see the level of the work done in the sample that is in the publications section.

The success of this course lies in the revelation that modern parents (who once were students) turned out to be good students of modern mathematics.

On October 31, 1967 at the Mackenzie University (San Pablo) foundations were laid for the first modern mathematics olympics intended for students of secondary school, with the aim of rewardin creativity and stimulating values in the field of mathematics.

We may bear in mind also that in the competitive entrance examinations for public school secondary teaching certification through which professors for secondary level instruction are recruited in the Province of San Pablo, modern content of erudition and methodology are demanded in the written tests.

Also, the current entrance examinations for students enrolling in the higher education schools (various specialities) in many Brazilian provinces require a modern mathematics content. In this way articulations are currently being developed between secondary level instruction and the enrollment of students in higher education in a climate of harmonious progress and in accordance with known results.

Conclusions

Much more remains to be done in the field of instruction in Brazil, notwithstanding the great deal that has been done in recent years and is being done at present.

But it is a reality that there is a progressive sentiment in mathematics instruction in my country and everyone feels it. It is not programmed progress. It is almost a "shotgun" progress based on the insistent idealism of a great number of Brazilian teachers who do honor to their country!

ACTIVITIES OF THE OAS IN MATHEMATICS

Andres Valeiras - O. A. S.

The OAS through its general secretary office, the Pan-American Union, has been contributing for some years to the efforts being made in Latin America to complete the education of professional mathematicians and improve mathematics instruction.

Originally the Division For Scientific Development undertook the responsibility for various programs; then beginning with the date of its creation July 1962 the Department of Scientific Matters put support to this action with its own means or in close cooperation with other departments of the OAS, state, or private bodies of various countries. Those of its activities which relate to mathematics have been directed towards the following goals:

a. Aid to Ministries of Education in their task of modernizing study plans.

b. Aid to Ministries of Education and Instruction Institutions to improve their inservice faculty and modernize their study plans for training of teachers;

c. Assistant to encourage research;

The activities undertaken to fulfill these aims include summer institutes in Latin America, exchange of scientists, regional meetings, scholarships for summer institutes and year round institutes in the U.S., a study of instruction in the sciences in engineering in Latin America (ECIAL) and various publications.

Mention must also be made of the importance of the action of the Department of Technical Cooperation through its various programs; these are the scholarship program, the university chair program, the special training program, the direct double aid program, the intregrated project program, and the technical cooperation program.

For purposes of clarity we shall consider first the general programs of the Pan-American Union which are assisted by the Department of Scientific Matters and subsequently the specific programs of this department developed independently or with assistance from other institutions.

The Scholarship Program

The scholarship program of the OAS is administrated by the Department of Technical Cooperation; fellowships are granted to citizens of countries which are members of the organization for continuing advanced studies in a high level institution in American countries other than those of the scholarship recipient. The scholarships may be for as long as 2 years. The first awards, 50 in number, were granted in 1958; by June 30, 1964, 2552 granted, of which thirty eight were in mathematics and fifty seven in statistics, although the latter were not all mathematical statistics.

Between the 1st of July of 1963 and 31st of August of 1966, 1688 scholarships were given of which 17.23% were designated for scientific and cultural work. Of these twenty four were in mathematics while fifty one in economic development and thirty seven were in statistics. In 1962, eight scholarships were given in mathematics and twenty in statistics.

2. Program Of Direct Technical Assistance

In this program consulting services and other help is granted to institutions by brief visits of experts, generally for about three months. The program was begun in 1959. Recently, seventy seven missions have been accomplished, of which none was in mathematics but sixteen were in statistics. Actually there has not been any mission of this kind in mathematics since the beginning of the program.

3. University Chair Program

Through this program funds are supplied for the establishment of visiting professorships in universities in the member countries. The position usually last for one year. The program was establish in 1960 and the first chair granted was in mathematics. Up to 1964, thirty seven chairs were granted of which four were in mathematics and one in statistics.

4. Special Training Program

This program was established in 1952 with the collaboration of UNESCO. It grants groups scholarships by organizing special training courses for the scholarship recipients in Latin American countries and giving individual scholarships to attend the courses. The courses are given in countries outside of the Americas, the host country undertaking part of the expense. Up to July 1964, thirteen such courses were given, one of which, in Spain with 9 participants, dealt with statistics.

A number of courses have been given to date, of which only one included mathematicians. This was a course for improvement in basic sciences, divided into four sectors for physics, chemistry, biology, and mathematics respectively. It is currently being given in Spain.

5. Technical Cooperation Program

This was created in 1951. In 1958 it was incorporated into the new department of Technical Cooperation. The objective of the program is to contribute to the economic and social element of American countries to the training of personnel at advanced levels in study centers especially created to advance national education and research institutions as much as possible.

To date 26 projects have been approved of which 13 are in operation. Among these is the Inter-American program for improving instruction in sciences, the only one of the implemented projects to date which relates to educational problems.

5.a) Programs for integrated projects

This was begun in 1965. By integrated projects we mean thos which endeavor to provide, according to the need and in a coordi nated manner, training, technical aid, equipment, etc. to govern mental agencies, institutions, and universities, in Latin Americ with the aim of effectively reinforcing their work.

These projects have the advantage of concentrating funds fo training and technical assistence in the institutions instead of dispersing efforts through scholarships and consulting, procedu which are more isolated and sporadic, making it difficult to maintain a framework and sustended pattern in time and environme

6. Inter-American Program for Improving Instruction in the Sciences (PIMEC)

This is one of the 13 projects of the Program of Technical Cooperation now in operation. It was proposed by the Department of Scientific Matters which is the cooperating body in charge of the administration and it was approved by the Economic and Social Council in 1963. It began operations in July 1964 with headquarters set up in Montevideo in April 1965.

Among the activities of the PIMEC which are of interest to this meeting the following may be cited:

a) Institute for University Mathematics Teachers:

This was developed in Montevideo between the 13th of Septembe and the 10th of December of 1965 with the participation of 32 professors from 13 different countries. Common courses were given for all the participants in algebra and introduction to set theory the former throughout the entire institute, the latter during the last six weeks. There was also optional courses in probability, statistics, and geometry during the first six weeks.

b) Algebra course for Mathematics Teachers

This was given in Montevideo between January 10th and April 1st, 1966, with the participation of 24 teachers from 11 countries. Other teachers attended as auditors without scholarships. During the first nine weeks, courses were given in algebraic structures and linear algebra, with the insertion of a section on Boolean algebra. During the last three weeks, workshops were given on applications of Galois Theory and linear algebra.

c) Course for Intermediate Level Teachers of Uruguay

This was a local two-week course during which concepts of algebra, analysis and probability and statistics were presented of the interest of secondary level teachers.

d) Course in Probability and Statistics for Biologists

This course was given at the request of Uruguayan Biology teachers presenting elements of descriptive and mathematical statistics as well as probability and statistics. No book was followed and notes prepared by the instructors were used; some of these will be published as monographs. In some cases, these courses are supplemented by lectures, individual work by the participants and the information presented by the students about the state of instruction in their respective countries.

In particular, for the second of these courses, PIMEC specially invited Professor George Papy who came to Uruguay and gave several public lectures.

e) In order to analyze the present state of intermediate level instructional programs in Latin America and adapt future courses and seminars to conditions, and to study suggestions for later modifications of courses, two inquiries were made to gather adequate information.

f) The Bulletin of PIMEC information is now being published.

In it, an attempt is made to gather all possible informati on initiatives and experimental results taking place in Latin America and in other countries to improve instruction in the sc and particularly in mathematics. In as much as most of the peop interested in these problems are gathered here, representing a responsibility to lead the efforts toward improvement of the function of mathematics teachers, I feel it important to call upon you to see to it that information about your work reaches so that it may be made known in Latin America.

7. Inter-American Center for Instruction and Statistics (CIENES)

The CIENES was created in 1962 by a transformation of the Inter-American Center for Instruction in Economic and Financial Statistics (CIEF), an institution in existence since 1953. The CIEF gave regular courses at its headquarters and regional courses in study centers in Latin America. In 1961 the CIEF granted 31 scholarships. The CIENES has its headquarters in Santiago, Chile, and is a dependency of the Department of Statistics. Since 1962 the CIENES has been giving three kinds of courses:

a) courses in statistical techniques for personnel of institutions that produce basic statistics, principally the General Administrative Offices of Statistics and Census or similar agencies. These courses usually last 10 months.

b) Courses in Economic and Social Statistics for personnel of institutions which formulate and collect statistics and make studies of an economic and social nature, such as the central banks, agencies for stimulating production, groups responsible for programming economical and social development. These courses last 10 months.

c) Courses in mathematics statistics for teachers of university level statistics and consultants on statistical methodology. These courses last two years in two 10-months periods.

In its first year, the CIENES had 41, 43, and 18 respectively in these courses. In 1966, 35 scholarships recipients participated in course a), 32 in b), and 25 in c), these last dividing into [illegible] the first year and 12 in the second. Of all the participants on scholarships it should be pointed out that 65 are CIENES scholarships, the others coming from other national and international institutions.

While the courses of type c) may appear to be particularly useful, I feel that mention should be made of the total picture of the work of the CIENES since it is an institution which has had considerable influence in the development of statistical studies in Latin America.

Let us consider now the special programs coming under the Department of Scientific Matters of the Pan American Union:

1. <u>Summer Institutes in U.S. Year-Long Institutes</u>

By virtue of an agreement with the National Science Foundation 20 scholarships have been given every year since 1960 to enable high school and university teachers from Latin America to participate in summer institutes sponsored by the NSF. Such institutes are held in American colleges or universities and may be for teachers of mathematics, biology, physics, chemistry, or applied sciences. Their duration varies from 4 to 12 weeks in general.

During the first three years, up to 1962, 56 scholarships were granted, of which 11 were for mathematics teachers. Between 1963 and 1965, 67 scholarships were granted of which 17 were in mathematics and two in statistics.

During the year 1966, 26 scholarships were granted of which four were in mathematics. These institutes have added meaning for participants by the fact that a number of them manage to extend their scholarships by participating in another full-year academic institute following the summer institute; in some cases masters degrees were obtained.

In conjunction with this plan, Latin American scientists were invited to become acquainted with the summer institute system through visits as lecturers.

Summer Institutes in Latin America

Since the beginning of 1961, summer institutes in sciences have been held in Latin American countries. Those involving mathematics were the following:

a) Summer Institute of Science for Central America and Panama. This was given in San Jose de Costa Rica, January and February, 1961, lasting six weeks. It was designed to deal with biology, chemistry, and mathematics. Total number of participants was 41. In 1963, similar courses were given with the participation of 138 teachers. The Pan American Union and the National Science Foundation of U.S. financed these institutes acting in collaboration with the University of Costa Rica, headquarters for the courses and for the USUCA.

b) Summer Mathematics Institue, Lima, Peru.

This was held with the collaboration of the Institute for the Promotion of the Mathematics Instruction of Lima, Peru, financed by the Pan American Union, the National Science Foundation, and the Ford Foundation. It was intended for teachers from teacher training schools and high school instructional programs. It lasted six weeks in February and March of 1962. Eighty teachers took part in it, 67 being Peruvian, 13 from 6 other countries.

c) Summer Mathematics Institutes, Lima, Peru.

This was similar to the previous one but was developed on two levels. In its organization, the Pan American Union, the National University of Engineering, the Ministry of Education collaborated, with financial backing from the National Science Foundation in the U.S. and AID; the participants were high school teachers and teacher trainee school teachers. 70 of them were Peruvian, 60 from other countries. The course lasted six weeks in February and March, 1964.

d) Summer Institute in Central America.

This was given in Costa Rica in 1963, and followed the lines of the 1961 institute.

3. Program of Exchange of Scientists

This was begun in 1960 through the cooperation of the NSF. Its goal has been to strengthen research centers and instruction programs in Latin America by stimulating the formulation and development of joint projects among groups of scientists and different institutions.

In the last four years, 46 scientists participated in the program, three of these were mathematicians. The projects last a semester or an academic year.

A similar program in collaboration with various institutions has enabled mathematicians and mathematics teachers from Latin American countries to become acquainted with the various projects for mathematics instruction improvement in the U.S.

4. Regional Meetings

The OAS has contributed to the financing and organization of various regional meetings on instruction in science, in particular, the first Inter-American Conference on Mathematics Instruction held in Bogota in 1961 and this second Inter-American Conference on

Mathematics Education which has brought us together. The recommendations of these meetings were published in a leaflet for distribution in Latin America.

Publications

The Department of Scientific Matters of the Pan American Union had been constantly concerned with the distribution of publications of interest to scientists and teachers of science. Publications issued to date referring to mathematics are the following:

1961: Guide to Scientific and Technical Channels in Latin America, of a general character.

1963: First Monograph entitled "Revolution in School Mathematics".

1965: Second Monograph on Vectorial Spaces and Analytic Geometry.

In December 1965, the special advisory committee met in Washington and proposed the publication of a series of 12 monographs on the following topics:

1. Functions
2. Fundamental Algebraic Structures
3. Linear Algebra
4. Fundamentals of the Euclidean Geometry from the point of view of Linear Algebra
5. Introduction to Topology (topology of metric or measured spaces)
6. Fundamentals of Mathematics
7. Introduction to Probability and Statistical Inference

8. Linear Programming
9. Real and Complex Numbers
10. Numerical Characteristics
11. Some Applications of Mathematics
12. History of Modern Ideas in Mathematics

The names of outstanding Latin American mathematicians have been proposed for the writing and editing of the series.

This program of scientific monographs is being financed with the help of the NSF, and at this moment some of the volumes are being readied for publication.

I should like to mention here the publication of numbers 37 and 38 of the Education Review (1965), a publication of the Department of Educational Matters of the Pan American Union, totally devoted to Mathematics Instruction.

6. Study of the Teaching of Sciences and Engineering in Latin America (ECIAL)

The ECIAL study is being done through the support of the Ford Foundation and seeks to bring to date the existing information on teaching of science and engineering in Latin America.

In the publication, corresponding to each country, data is given on school organization, high school and university study plans, the syllabus and bibliography for each course, indication of post-graduate courses and fields of research and publications.

To date we have published the reports for Chile, Central American Countries and Mexico; those for Argentina, Brazil, Colombia, Uruguay and Venezuela are in process, and the remaining reports will be published in the latter part of 1967.

7. Guide to Scientist and Scientific Institutions in Latin America

In collaboration with UNESCO, guides for scientific institutions and scientists were published for five countries. These publications bring up to date those published seven years ago.

Conclusions

The foregoing has been a sketch of the activities of the OAS in recent years contributing to improvement of mathematical instruction in Latin American countries. The effort has had several orientations of which the most important may well be the very useful effort to awaken interest in the improvement and to use the analysis for suggesting lines of future action.

The OAS hopes that from a meeting of this sort concrete suggestions will result. They will certainly be taken into account, the only qualitative restriction being financial resources available to support the necessary activities.

C. On Curriculum and Transition

AN EXPERIMENT IN RECONSTRUCTING THE CURRICULUM FOR SECONDARY SCHOOL MATHEMATICS

Howard F. Fehr, U.S.A.

It is quite commonplace to talk about modern mathematics, modern methods, and a new mathematics program as if what we are trying to do in the schools today is greatly at variance with what we did ten or twenty years ago. It is a fact, however, that in spite of some high-sounding changes that have recently been instituted, what we are doing today in mathematics does not differ much from the traditional college entrance program of forty years ago. Algebra is still studied for $1\frac{1}{2}$ to 2 years as a separate branch isolated from the study of all the other branches of mathematics. The study of geometry has been reduced to one year, but it is still, for the most part, Euclid's synthetic way of doing geometry.

By no means do I wish to deprecate the efforts and improvements we have made, for they are a big step forward. But it is only the first step toward the type of mathematical education needed in the world today. We now see more clearly than ever before the difference between updating or modernizing a traditional program and the total reconstruction of an entire curriculum. There are very strong reasons for desiring a totally reconstructed curriculum.

In making a modern syllabus, the first step is to select a set of major topics, using some agreed upon criteria for making the selection. If all prejudices are abandoned, this first step will result in the adoption of new topics as well as the rejection of certain others, which have been entrenched in our curriculum, This selection must be made by persons highly competent in mathematics, for they alone have sufficient knowledge to know the essential and necessary elements that should be included in a modern mathematics program. Once the topics are selected, they must be arranged in some suitable sequence. However, orderings will proceed from the point of view one takes Depending on whether one is concerned with:

(a) a rational arrangement of topics,

(b) the urgency of learning and applying the topics, or

(c) the process by which the mind comes to comprehend the topic, one can consider respectively a logical, a practical, or a psychological order. It is essential, that without requiring too great a compromise all three orders should be reconciled into a good pedagogical sequence.

Today, we are under great pressure to teach more mathematics to students at an earlier age because of the steady growth in the applications of mathematics. At the same time we are more sensitive to the psychology of learning related to mathematics about which, however, our knowledge is limited. These factors make the problem of compiling a mathematical syllabus a very complex one. On the other hand we have a greater understanding and deeper insight into the concepts and theories of contemporary mathematics. It is this aspect of modern mathematics which places us in a good position to produce a unified syllabus.

To achieve our objective, it is not enough to get rid of obsolete subjects, nor to replace them by subjects of a more modern variety, nor to graft a few modern concepts onto an outdated program. School Mathematics must be reconstructed by making use of mathematical structures.

A Proposed Syllabus

In building a genuinely modern program, we must consider a certain number of key ideas which are essential as unifying elements, and then add others that are of value because of their extensive applications, both pure and applied. In this context, sets, mappings, relations, and functions are fundamental to the study of all mathematics - they are unifying elements. Basic to all secondary school mathematics are the algebraic structures, group, ring, and field, and the algebraic-geometric (or geometric-algebraic) structure of vector space and linear algebra. The calculus, probability and statistical inference, and some mathematical theory relating to computers provide a fitting and satisfying climax for the seoondary school program. At all times instruction should be centered on and headed toward the development of vector spaces.

Not only do these topics provide for the needs of those who will use mathematics, but most significantly they provide the substance for a complete general or liberal arts mathematic education which the secondary school must provide in the future.

It is necessary to adopt a sensible and responsible attitude in proposing any syllabus to see that what is proposed can be done properly, with unhurried calm, and with complete understanding. The underlying assumption of the experimental proposal outlined below is that mathematics organized functionally, under broader and more general concepts, based on the acquisition and use of pervading structures, and taught with clever and psychologically sound pedagogy can achieve in a shorter period of time than heretofore, more mathematical learning of a contemporary and useful nature.

In interpreting the emphasis that has been given to structure one may be apt to think it refers to axiomatic or postulational structure and rigor. This is not intended at all. In the elementary and secondary school, a formalistic-axiomatic study of mathematics does not provide a medium in which freedom of the mind can be developed. Logic discovers nothing - and in its proofs everything must be foreseen. In real learning discovery always comes before proof.

The Subject Matter

In giving a synopsis of a mathematics curriculum that is genuinely modern it is frequently necessary to talk about topics or branches in such a manner that they may be construed as constituting a year, or half-year, of sequential study. This is not the intent of this presentation. All the topics are to be introduced early in the program and extended, broadened, and deepened in the succeeding years of study. So we consider only the scope, or the amount of and type of mathematics that constitutes the program. No sequence is indicated here, and in fact there can, no doubt, be a number of quite different sequences, each efficient and effective in obtaining the goals described before. These sequences must be obtained by classroom experimentation in subject matter organization and pedagogical procedures.

I. Sets

II. Relations

III. Functions

IV. The Set of Cardinal Numbers

V. The Line and The Plane

VI. Groupoids and Groups

VII. The Ring of Integers

VIII. The Line and the Ordered Field of Real Numbers

IX. Numerical Calculations

X. Polynomials with Real Coefficients

XI. The Vectorial Plane and Affine Geometry

XII. Euclidean Metric Geometry of the Plane

XIII. Descriptive Statistics

XIV. The Plane and the Field of Complex Numbers

XV. A Formal Study of Groups, Rings, and Fields

XVI. Vector Spaces (Linear Dependence, Matrices, Solutions of systems of Linear Equations).

XVII. Affine Coordinate Geometry (See Levi)

XVIII. Euclidean Geometry of Space (by coordinates and vectors

XIX. Probability of Finite Sample Spaces

The Twelfth Year should include:

XX. Metric Space and Simple Topology (open spheres, neighborhoods, distances)

XXI. Continuous Functions (Continuity at a point, over an interval). The Limit of a Function at a Point.

XXII. Differential Calculus

XXIII. Integral Calculus

XXIV. Simple Differential Equations. Probability extended.

Organizing the six-year syllabus:

It may be of interest to see how the experiment to achieve the newly constructed syllabus was initiated. First a small group of mathematicians, acquainted with the past and present reform efforts in the U.S.A. and Europe were assembled to prepare a conference on the syllabus. This group stated certain rules of action and assigned two small sub-committees to prepare a working paper involving the philosophy, the subject matter, and the point of view with regard to instruction in Geometry. The tw position papers were amalgamated into one, with a flow chart of content and grade placement and distributed ahead of time to all the writers and mathematical consultants (for a three-week conference) as a position that should be accepted for the purpose of developing the syllabus.

At the conference of 8 U.S.A. mathematicians, 4 European mathematicians, and six professors and teachers serving as writers, the position paper, with slight modifications, was accepted the first day. The committee then sub-divided into committees on algebra, geometry, probability including statistics, analysis, and numerical mathematics - each to derive a six-year program in the particular field - and to show its inter-relations to the other fields. After eight days, with a few plenary sessions, the syllabi were accepted, and the first year secondary school work in each delineated.

Again a sub-committee was given the charge of flow charting the agreed upon syllabi into a comprehensive six-year program. Each of the original sub-committees were asked to develop in extended form the syllabus for the first year, with suggestions on organization and teaching the material. At the end of the conference the special committee presented the comprehensive unified syllabus which follows.

Although the topics are distributed among the six secondary school grades, only the first year program is definite at this point in the experiment. Both grade placement and order of topics in later years must be discussed and decided by later planning conferences. In fact, even the first year program will be subject to change on the basis of experience with it.

<u>Ordering of Topics in First Year</u> (Seventh Grade) - SSMCIS

0. Planning a mathematical process - introduction to flow-chart Reexamination of properties of (N, +, <).
1. Clock addition and multiplication, mappings and operations in finite sets.
2. Properties of operation in sets.
3. Mappings and transformations of N, Q^+.
4. Introduction to (Z, +, <).
5. Lattice points in the plane.
6. Reexamination of multiplication in N and development of (Z, +, ., <). Comparison of Z with (Z, +, .).
7. Mappings and transformations of Z^2.
8. Sets and binary relations on sets.
9. Transformations of the plane. Orientation in the plane.
10. Angular regions and their measures.
11. Elementary number theory.
12. Probability and statistics.
13. Construction of (Q, +, ., <).
14. Mass-point geometry.

15. Applications of Q.

16. Incidence and order - a small axiomatic system.

Topics in Second Year (not ordered) - SSMCIS

1. Sets and groups.
2. Axiomatic treatment of affine plane geometry.
3. Fields and introduction to the real numbers.
4. Perpendicularity, scalar products, and the Pythagoream Theorem.
5. Combinatorics.
6. Transformations in space.
7. Real functions.
8. Statistics; central tendency and dispersion.
9. Elementary trigonometry.
10. Axiomatic treatment of measure of plane sets.

Topics in Third Year (not ordered) - SSMCIS

1. Structure of affine three dimensional space and metric space.
2. Introduction to matrices, vector spaces, and solution of linear systems.
3. Study of the sphere.
4. Description of some real functions - including polynomial, rational and circular.
5. Axioms for volumes, geodesics.
6. Definition of the real numbers as a complete ordered field. Consequences of this definition.

7. Study of squares and square root functions.

8. Basic circular function formulas: $A \pm B$, $2A$, $\frac{1}{2}A$, "identit

9. Probability

10. Topology

11. The regular polyhedra and groups of isometrics.

12. Logarithmic function and slide rule.

Topics in Fourth Year (not ordered) - SSMCIS

1. Axiomatic treatment of two dimensional affine geometry. (Artin, Stone, Choquet, or Levi)
   Axiomatic treatment of metric plane.

2. Real numbers
   - Review definition of R.
   - Characterize sub-systems N, Z, Q.
   - Cardinality
   - Induction principle
   - Study equations of first and second degree.
   - Extend R to C in connection with quadratics.

3. Real functions and their graphs.
   - Review polynomial functions and their graphs and begin informal differentiation.
   - Investigation of $x \longrightarrow 1/x$.
   - Study of graphs of: $f(x) + a$, $f(ax)$, $af(x)$.
   - $Ax^2 + Bx + C$.

4. Continuity in metric spaces, limits, convergence of sequenc
   - Examples of metric spaces.
   - Continuity of a function at a point.
   - Convergence of sequences and series.

5. Probability.

Topics in Fifth Year (not ordered) - SSMCIS

1. Groups, rings, and fields.
   - Groups and subgroups.

   - Algebraic extension and polynomial functions.

2. The complex field.

   Extension from R to C.

   Argand plane with geometric interpretation of operations in polar coordinates - DeMoivre's Theorem.

   $z \longrightarrow az$, $z \longrightarrow (az + b)$, $z \longrightarrow z$, etc. with geometric interpretations.

   $z \longrightarrow z^n$. Riemann surfaces

3. Continuity and limits of functions.

   Differentiation

   Derivatives of sin x and cos x.

4. Integration

   Definition and properties of integral.

   Applications.

5. Study of logarithmic and exponential functions.

   Definition and study of e^z with geometric interpretations.

6. Simple differential equations.

7. Affine image of circle and some properties of conics.

8. Probability

9. Statistics

<u>Topics in Sixth Year</u> (not ordered) - SSMCIS

1. Vector spaces and groups of affine transformations.

   Metric properties of a scalar product.

2. Linear algebra.

   Linear mappings of vector spaces.

3. Algebraic structures

   Rings and domains.

Fundamental theorem of algebra.

4. Analysis

Mappings from R into a vector space including review of differentiation.

Taylor formula, power series, redefinition of circula and exponential functions, consequences.

Linear differential equations of second order with constant coefficients.

$\sum_{u \in A} p(u)$ where A is denumerable and $o(u) \geq 0$.

5. Probability and Statistics. (Grade 12)

<u>Subjects for Seminars and Independent Study</u>

1. Angle measure.
2. Measure theory.
3. Normed vector spaces
4. Quadratic forms.
5. Frobenius theorem, Galois theory, Boolean algebra.
6. Polynomial theory.
7. Determinants and inner product vector spaces.
8. Numerical methods for solution of differential equations.

The 1st year book (Grade 7) has been written and is being taught by twenty special prepared teachers to 350 selected good students. After three months we know the pupils can do the wor outlined for the First Year in 0, 1, 2, 3, 4, and 12, above wit genuine success. After two more years we shall be able to see if the rest of the program fares as well.

MATHEMATICS PROGRAMS IN THE TEACHING OF ENGINEERING

Carlos Imaz

Not so long ago in Mexico City, at the ninth Convention of the Panamerican Union of Engineering Associations, Professor Ralph A. Morgan of the Stevens Institute of Technology spoke about the general problems of engineering teaching and viewed the subject from two angles, the first referring to "developed" and the second referring to the "emerging" countries. I assume that Professor Morgan has no ulterior motive and understands this differentiation from the point of view of technological and economical development in the main; and it seems to me that this division is essential since "developed" countries have reached that state, even where it is to a small degree, by having more effectively solved the type of problems which concern us here and which are more acute and critical and therefore more in need of attention in the "emerging" countries, particularly in Latin America. This justifies our analyzing the matter from the view point of the aforementioned division of countries.

For many reasons, most of them well known, the prevailing solution is the one that suggests itself most readily; that is, copying what is done in the more advanced countries and applying it more or less directly in each particular case. I am not suggesting at all that we can not profit from the experience of others; but rather that we must first of all carefully analyze our own situations in order to be able to determine what portion of that experience is useful for us and what should be replaced by solutions that are particularly appropriate to each case.

Let us get down to specifics. One thing is clear and incontrovertible: mathematics plays a basic role in engineering, and therefore mathematics instruction programs are needed for those who study engineering. But how shall these programs be formulated? One way that is not too laborious consists of asking engineers about the mathematics they need and basing our program on an average of the data obtained. Apparently the great majority of programs presently in use originated in this prosaic way. How do these programs fare? Like most "averages," they are not adjusted to any one. For many, and in our case for the majority, they turn out to be too broad; for the rest, they are insufficient. Some may think that this situation can easily be rectified later, since the former group forgets what they were taught (optimistically assuming that they once learned it) through the simple experience of not using it; and the others acquire the necessary knowledge as they need it. From the point of view of modern engineering, such a procedure is totally unacceptable. But this is not all; this view forgets to take into account the available human resources for carrying on instruction, the relationship of instruction to the other disciplines that are part of the engineering curriculum, and almost entirely the various special aspects that should characterize mathematics instruction for engineers asagainst instruction for other groups. And finally, no account is taken of the actual prior education which the students have had by the time they enter the School of Engineering.

These and other reasons make it clear that the mathematics curriculum of Latin-American engineering schools should be changed, taking into account two principal objectives: the incorporation of the new directions and advances to better utilize human equipment, professorial as well as student. We must bear in mind that the effective implementation of these changes is not a new and isolated academic fact but rather represents, undoubtedly, one more step that must be taken for the rest of the "emerging" countries to have hopes of adequacy; that is becoming "developed" countries, some day.

As I pointed out earlier, one of the most obvious defects of the mathematics curriculum in our engineering school is that, for many, it is too extensive. This generally is based on the feeling that mathematics is what makes the engineer. This might be true for some very special kinds of engineers, but even then I have doubts, since while it is true that mathematics is an indispensable instrument, it is also true that mathematical methodology is not applied in engineering and is therefore only a part of the education and formation of engineers.

The problem of the extension of the mathematics curriculum is quite serious. In the great majority of our universities, engineering degree programs are studied along fixed lines (gradually, in five years), to which all students are subjected, regardless of the kind of professional activity they intend or are able to go into. That is, a routine technical job, advanced technical work, or instruction and research. And since there are no technical schools to train people for the first of this kind of work, we are left with the fact that people interested in studying any kind of engineering must go through the same single type of instruction.

To show beyond a doubt that this exaggerated extension of mathematics programs does exist, I shall present some comparative data. Let us take the basic curriculum for engineers, Which should be entered with 350 class hours according to the recommendations of the Committee on the Undergraduate Program in Mathematics (CUPM). In Soviet Universities, for example, in the civil engineering major the mathematics program occupies 400 hours (including both theory classes and exercise). In Mexico, any engineering major includes about 640 class hours in mathematics, besides extra classes for exercise.

In Mexico as well as in other Latin-American countries, this situation creates a series of serious problems. In the first place it creates an enormous demand for teachers who must be trained for a rather high level of mathematics. This demand is not close to being met, and I doubt that it can be met if the situation continues along the same lines as the present; since among other things, there might possibly be no solution to the problem of having the necessary number of well prepared teachers.

We also must face the undeniable fact that these mathematics courses constitute great obstacles for the students. This can be fully verified by observing the rate of drop-out. For example, in Mexico approximately 70 percent of the students who enroll in engineering schools discontinue their studies during the first two years, because they are not able to satisfactorily pass the aforementioned courses. The worst part of this is that in this time the students have received no technical education and leave the university no better off than when they enrolled and also with a clear feeling of frustration. The injustice and inefficiency of this situation becomes even more obvious when many of the students who do succeed in continuing their studies later discover that the later courses in the major are completely heuristic in form and in most cases use simple introductions to handling manuals which hardly ever use any of the mathematics instruction which was previously received and which, from disuse, is forgotten by the time the degree program is completed. This is so generally true that those students who later undertake graduate studies have to be taught again the mathematics that they were assumed to have learned in the undergraduate degree program.

But the worst of it all is that things continue forward. Today it is said that the engineer constantly needs greater knowledge of mathematics. This is true for certain kinds of engineers but it is causing an increase in the mathematics material they attempt to provide for all engineers, which consequently worsens the situation we have been discussing.

At best, if this situation continues, it can result in the education of a small number of highly qualified engineers who are, of course, absolutely needed, but who will find themselves in a social milieu in which they can not discharge the duties for which they were trained.

This leads to one of two things, either the engineer must devote himself to tasks requiring less than the high education he has received, or else he emigrates to places where he can utilize his professional capacity more completely.

It is often said that in Latin-America we have a strong tendency to seek the best directly, with no care for the intermediate steps of the process. When an engineering school is opened it is modelled after, let us say, the Massachusets Institute of Technology, for example. The point is brought home most clearly by pointing out the situation in a different field, such as Medicine. The medical school in Mexico is world renowned, yet curiously enough there is not one reasonably acceptable school of nursing. As a result, about the only thing doctors do not do in hospitals is wash the floors.

Of course, I do not believe that the conditions I have pointed out are the only ones involved in setting up mathematics programs for engineering majors. But I am fully convinced that these conditions are generally ignored, and for that reason I have gone into them somewhat in detail.

I have the impression that the road that should be followed for the formulation of mathematics programs in engineering schools can now be seen quite clearly. In the first place, in those instances where the situation is that described in the previous paragraphs we should promote the restructuring of the engineering schools to make them sufficiently flexible to produce engineers at the three levels of activity mentioned. At the same time--and it should be the job of combined committees of engineers and mathematicians--the mathematical needs for each level should be analyzed. This analysis must be made with strict attention to the real needs and avoiding any mere intention of providing erudition. Unless things are handled this way, what will happen will be what has just happened in Mexico; that is, that the increased need for mathematics for high-level engineers has resulted in an even greater broadening of the new programs and a consequent worsening of the situation. For example, there are barely any teachers to give these courses. Yet this is not the worst of it; the single strainer through which all engineers must pass has been made even finer.

This does not meet the real needs of our technological development. While I do not pretend to know exactly the mathematical needs of the different engineering programs, I do wish to be a bit more specific and offer some idea of the possible structuring of the mathematics curriculum according to the previously indicated levels. This we shall call first, second, and third, in ascending academic order.

For the first level it seems to be sufficient, in practical terms, to provide a knowledge -- more manipulative than conceptual-- of algebra, calculus and analytical geometry. By this I mean that although any mathematics course for engineers, at any level, should be characterized by "motivation," "intuition," "thoroughness," and "application," the thoroughness should not be used at this level to select engineers. It seems absurd to me, for example, to dismiss people because they can not prove the theorem of the mean in differential calculus. The "thoroughness," should be used with an eye-dropper and only where it is clearly necessary and not in matters that are rather apparent. For example, I know by experience that it is completely useless to prove things like the continuity of products of continuous functions with rigorous arguments of the ϵ and δ type.

On the other hand, the aspects of "motivation," "intuition," and "application" are the aspects that should receive preferential handling in these courses. At all costs, we must avoid students being given only empty (at least for them) mathematical definitions, which happens more often than one might believe.

We might add to these basic courses others such as statistics and probability (elementary), notions of programming and numerical analysis, graphic methods, etc. For example, the algebra course might be a good place to teach elementary programming and if a simple table-model calculating machine is available, numerical problems can be outlined to be solved by the students -- a procedure which generally stimulates their interest and enthusiasm.

The situation at the second level is considerably more complex and surely will depend a great deal on what particular branch of engineering is involved.

What we can do is designate subjects that will indicate the level of mathematics knowledge that it would be desirable to achieve, repeating that in these courses also emphasis should be placed on the stages of "motivation," "intuition," and "application." The curriculum at this level can include among other things subjects such as advanced calculus, differential equations, numerical analysis, equations with differences, complex variables, differential geometry, etc., according to the particular needs of each major. Finally, in reference to the third level, it is practically impossible to assign boundaries, but this would surely be the level for topics like integration theory (Lebesque), advanced courses in differential equations, integral equations and other special topics of fundamental analysis, etc.

I must emphasize that throughout this curriculum we must be aware of the degree of sophistication in the teaching and not let these programs become theoretical and highly impractical, which brings me to the absolute importance of having completely competent mathematics professors (at least the Master's Degree) at all levels. They alone can properly dispense mathematical knowledge. From which we infer that mathematics schools face another important task, that of producing these teachers. This may seem obvious to many, but the truth is that it is overlooked with alarming frequency and especially in those places where the so-called pure mathematics is more developed, since in these instances there has been a polarization of professorial interest (and therefore student interest) toward the most abstract and advanced aspects of mathematics, with a consequent neglect of the intermediate sector. What happens here resembles what I pointed out in the beginning with respect to engineering schools. Our fine science schools are capable of producing excellent mathematics researchers who in many cases go off to swell the academic ranks of more advanced countries; yet they are not able to produce more teachers of advanced calculus. All of this originates in large measure from a desire to adopt the false principal that publicational research is a necessary condition for teaching, a principal which in many places is forcing the academic world into the attitude of "publish it now, think about it later, and in the meantime teach your classes." Another characteristic of the mathematics courses for engineers which I believe must be changed is its concentration. The usual tendency is to fill the students with as much as possible in one or two years and then never return to the topic. This has two disadvantages: on the one hand the danger of indigestion and thereby the opening of a dangerous gap between initial studies and graduate studies.

Nevertheless, it is clear that the study of possible distribution is a delicate matter that must be looked into with care.

To conclude, I wish to mention another level of engineering that I have not touched upon here. It is the level of the so-called "mathematician-engineers" or "applied mathematicians" which is undoubtedly of great importance and should be studied. But it is my conviction that these people should be basically trained in the schools of sciences and not in engineering schools and therefore should be taken into account in the reconstruction of the programs of the former institutions.

THE DANISH MATHEMATICS PROGRAM

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Introduction

This account is primarily concerned with the mathematics teaching in the Danish high school, the so-called "gymnasium" (10. - 12. school year). Besides there will be given some indications of the mathematics teaching at the universities, particularly with reference to the education of high school teachers.

The Danish "gymnasium" is a selected school. It is difficult to give a precise precent of the population which attends this school, but this percent is increasing from year to year. It is reasonable to say the mathematics line of the "gymnasium" is recruited from the top 20 percent of the population (These figures refer to intelligence).

The "gymnasium" is divided into several branches. At the moment we have two linguistic and three mathematical branches. The most advanced course in mathematics is given in the mathematics-physics branch (approximately 6 hours a week in each three years). In the following I will concentrate on this course.

All the Danish high schools follow the same study plan given by the ministry of education. But the choice of textbooks is left to the teacher.

The study plans for the mathematics-physics branch include two foreign languages (+ Swedish and Norwegian). Thus the Danish "gymnasium" is a rather broad school with no extreme specialization in any subject. The Danish plans were written in 1959-60 and have been in official use since 1963 (at a few schools since 1962).

Thus the theme for this report is: How do some moderately modern study plans look after 4-5 years' of use? In which directions did they go too far and in which directions were they too moderate?

The authors of the Danish study plans were very cautious. They did not introduce subjects which were too difficult for students of the expected standard. But, these plans were probably too extensive. It required great self-discipline to exclude important topics which are interesting and well suited for the teaching. But it was necessary to do so in order to secure sufficient time for thoroughness in the treatment of subjects in the study plans.

An important part of the picture of the mathematics teaching is the teachers' scientific and pedagogical background. The Danish high school teachers' education consists in a university study (normally 5-6 years) resulting in a masters degree, and a smaller course in theoretical and practical pedagogy (1/2 year).

At the university the future teachers get the same education as the future scientists. There is no distinction between these two categories of university students, and very often the students do not decide their professional career until the end of their study. Thus the high school teachers have a very good mathematical education. But I want to emphasize that they are in no way over-educated. It requires an extensive scientific insight to give valuable teaching to gifted pupils in the age group 16-19 years. It can be mentioned that the introduction of the new study plans met considerable difficulties in the beginning, primarily due to the fact that the teachers' background was too weak with respect to modern mathematics.

Mathematics in the Danish High School (Math-Physics Branch)

The central topics in the high schools in Denmark are analysis (differential and integral calculus) and geometry (esp. analytic geometry). The new study plans have added "abstract" algebra as a third main topic. In itself, this third topic is of very modest proportions but it has a strong influence on the treatment of the other topics.

Analysis

The old study plans (before 1960) contained a rather rigorous treatment of continuity, differentiability and integration. The new study plans do not contain any explicit demands for a modernization of these subjects but here (as everywhere) the study plans are flexible such that there are good opportunities for the teachers and for the textbook writers to let modern ideas influence the presentation.

In most of the schools the treatment of continuity is based on topological concepts, even though these concepts are not introduced in a systematized way. Continuity is defined formally only for real functions and for mappings between metric spaces. But the definitions are formulated in a language which is easily adapted to more general classes of mappings. The experiences with this presentation have been favourable. Continuity is more immediately understandable when it is linked to the concept of neighbourhoods than when it is presented in a pure ε,δ-language. On the other side, I do not feel any need of a more general treatment of continuity in the school. The students who will study mathematics at a university are well prepared for a course in topology. The other students have a fairly accurate impression of topological ideas without being burdened with technicalities.

Also the treatment of differentiation and integration can profit from newer concepts and ideas. Let me, as an example, mention two possible definitions of the derivative $f'(x_o)$:

1) $\Delta f(h) = f'(x_o)\cdot h + o(h)$

and

2) $f'(x) = \lim\limits_{h \to 0} \frac{\Delta f(h)}{h}$

The first definition is easily generalized to mappings between finite dimensional vector spaces,

$$f : R^n \longrightarrow R^m$$

while the second definition only applies to mappings where n = 1.

Naturally this is not a decisive argument for using definition (1) if we only consider functions of real (or complex) varialbe. But I have the impression that definition (1) is at least as easy to grasp as definition (2). And it has certain advantages in proofs and in applications. Also it removes the air of mystery which surrounded the differential df in certain older textbooks.

One of the catchwords in our modernization of mathematics teaching is "applications of mathematics." In Denmark we have not gone very far in this direction. We have not found much time to spend on other applications than the traditional ones. But in this connection I want to mention that Taylor's formula is now a part of the syllabus. It is easy to prove and it gives good opportunities to show other applications than the purely geometrical. On the other hand, unfortunately, the Danish study plans do not include differential equations. Apparently linear differential equations of first and second order is an obvious subject for the high school.

I can summarize my remarks about analysis by saying that knowledge of newer concepts and ideas can be of advantage even in relation to a rather conservative course in calculus. This knowledge should exist primarily in the mind of the teacher. It is less important to have high school students learn a completely developed "modern" ("abstract") treatment of analysis.

Geometry

The old study plans in geometry included plane analytic geometry with a detailed mention of parabola, hyperbola and ellipse and their geometric properties, some trigonometry and a little about geometrical construction, as well as some solid geometry.

The treatment of the conic sections and the trigonometry has been reduced considerably and the geometrical constructions have disappeared. The study plans prescribe that the treatment of geometry should be based on the concept of vectors but it is not required that the concept of a vector space be mentioned. The study plans do not mention affine geometry but a treatment of certain affine mappings are demanded. The majority of the Danish high schools arrange the geometry course in the following way:

1. year (10. school year): The vectors are introduced (intuitively) as equivalence classes of directed line segments, based on a rather traditional geometry course in 5.-9. school year. It is shown that the set of vectors has the properties of a vector space with scalar product, and the treatment of the plane analytic geometry and trigonometry is based on these properties (the word "vector space" is not mentioned in this year.).

This school year begins with a (purely intuitive) mention of elementary set theoretical concepts and terminology. Besides this and geometry there is a treatment of equations and inequalities. Furthermore a first treatment of fundamental algebraic concepts (composition rules, groups) and also a little number theory.

2. year (11. school year). The major part of this year's work is devoted to analysis.

The only geometric topics in this year are affine mappings of the plane and a little about conic sections (partly in connection with affine mappings).

The treatment of affine mappings is mainly analytic and the description of affine mappings by 2 x 2 matrices is studied. (Matrices is not a compulsory subject in the Danish gymnasium but it is advantageous to introduce them in connection with their geometrical applications. We only consider 2 x 2 matrices but this is sufficient in order to give an impression of the principal ideas. The course is undoubtedly a good preparation for a later course [at the university] in linear algebra.)

Certain important groups of affine mappings are studied in particular (e.g. isometries, similarities). Under these investigations the students get ample opportunities to use their (rather modest) knowledge of group theory.

3. year (12. school year): The geometry in the third year consists of an axiomatic treatment of solid geometry based on the axioms for a n-dimensional vector space with scalar product. Naturally a great part of the theory is restricted to n = 3. But experience has shown that it is not necessary to make this restriction. The students find it quite natural to treat geometry axiomatically in this way, and some of the students are actually a little disappointed when the restriction n = 3 is made.

This treatment is not demanded in the official study plans and there are schools where the solid geometry is treated in the traditional way. But the axiomatic treatment is not in disagreement with the study plans.

Algebra

The fundamental algebraic concepts (composition rules, groups, rings and fields) form a new topic in the Danish syllabus. The official study plans are very moderate with respect to this subject and the actual teaching has also been a little too modest. One should get a little beyond the mere trivialities when one introduces a new subject. More important than many purely algebraic theorems is the desire that the students get a thorough understanding of the fundamental concepts. Algebraic concepts and ideas appear so often in the treatment of other topics (particularly geometry) that the students do not consider algebra as an inferior subject.

In principle, it is reasonable to treat the algebraic concepts without going far in the theory. It is doubtful that proper group theory (apart from the most elementary parts) belongs in the school, not because the theory is very difficult, but because it is difficult to motivate the study and to exemplify the results. The primary value of algebra in the school lies in the applications to other branches of mathematics. Thus one should go no further than to the fundamental theorems about the kernel of a homomorphism.

For the benefit of other mathematical subjects the algebraic concepts should be introduced not later than the tenth school year so they are at hand when needed. There might be pedagogical reasons for an earlier introduction. It would be a great advantage in teaching in high school if the most elementary algebraic concepts as well as the elementary set theoretical concepts were introduced before the tenth school year. These concepts could probably enrich the teaching in the lower grades.

So far the treatment of groups has been too meagre in the majority of the Danish high schools, but there are reasons to believe that the algebra course will be improved in the coming years. Very soon the course will probably run along the following lines:

1. year (10. school year): Composition rules and their properties, groups, subgroups to a group. A first description of the algebraic properties of the numbers (this description is continued later in connection with the treatment of rings and fields). Groups of remainder classes, Cyclic groups. Isomorphisms and homomorphisms. The kernel of a homomorphism with applications.

Applications to number theory (Fermat's little theorem, elementary theory of divisibility in close connection with the study of the additive group of integers). Description of logarithm functions as order-preserving isomorphisms from the multiplicative group of positive real numbers to the additive group of real numbers.

2. year (11. school year): This year brings no new concepts in algebra. The students' understanding of algebra is widened through its applications in other subjects, particularly in the theory of affine mappings.

The description of the real numbers is extended when continuity of real numbers is discussed as an introduction to analysis. Furthermore, it is proved that the set of rational numbers but not the set of real numbers is denumerable.

year (12. school year): The algebra course in the twelfth school year covers polynomials, rings and fields, complex numbers. The study of rings and fields is primarily motivated by the investigation of the algebraic properties of the numbers, but also other rings are considered (e.g. polynomial rings, rings and fields of remainder classes). The set of real numbers is characterized as a continuously ordered Archimedian field, but there is no attempt to indicate a construction of this field.

The algebra course concludes with the construction of the field of complex numbers (based on the field of real numbers) and an investigation of the geometric and the algebraic properties of the complex numbers. The fundamental theorem of algebra is formulated with an indication of a proof, and some of its consequences are shown. The fundamental theorem is not mentioned in the official study plans.

It might be possible to introduce the complex numbers earlier than in the twelfths grade. But in Denmark we postpone this introduction for the sake of the physicists who insist on having differential and integral calculus as early as possible. An argument for a late introduction of the complex numbers is the very colourful introduction that it makes possible. And at this time the students have reached a sufficient maturity to appreciate a chapter which brings so many different mathematical topics in relation to each other (algebraic concepts, vectors in the plane, trigonometry, affine mappings, polynomials, etc.)

Other Subjects

In the official Danish study plans you find the words combinatorics and probability. But the demanded course is so small that it is not worth talking about. It might seem surprising that we neglect probability theory considering its still increasing importance. The only thing to say is that if we should give a reasonable course in this subject we would have to sacrifice other topics.

Naturally the elementary set theoretical terminology is required in the Danish gymnasium. But the study plans do not recommend a systematic course in set theory. At the moment we have to begin the teaching in 10. school year with a short survey of the set theoretical concepts and terminology, but there is reason to believe that within a few years these concepts will be introduced in the lower grades where they rightly belong. The only non-trivial set theoretical arguments in the gymnasium are the investigation of denumerability and Cantor's diagonal proof of the non-denumerability of the set of real numbers. It could be argued that we could omit these investigations since the results aren't used in the other subjects in the gymnasium, but we do not like to keep these beautiful ideas from the students.

Mathematics in the Danish Universities

The high school teachers receive, as mentioned in the introduction, their scientific education at one of the universities where they get the same training as the future scientists. The university education is supplemented by a limited course in theoretical and practical pedagogy. The practical part of this course takes place at a high school and is directed by teachers at this school.

Contrary to the high school education, the university studies are highly specialized. Usually the students study at most two subjects, a principal and a subsidiary subject. The subsidiary subject to mathematics is normally a science with close connection to mathematics, usually physics. Other possible subsidiary subjects are statistics and economy. It is also possible to study mathematics without studying any other subject.

The university education is divided into two parts. The first part (3 to 3.5 years) covers a series of fundamental mathematical topics, while the second part (2 to 2.5 years) consists in more advanced studies of a special topic chosen by the student. The study of the subsidiary subject is normally accomplished during the first part of the study. Mathematics as subsidiary subject is equivalent to the first part of the study of mathematics as principal subject.

The universities are more free than the high schools with respect to study plans. For practical reasons they use almost the same plans in the first part of the study in order to make it easier for the students to change from one university to another.

The study plans at the Danish universities do not differ much from the study plans in other countries. Thus I will restrict myself to give a broad outline of the subject list for the first part of the study. (This list is arranged according to topics, not according to time).

Advanced Calculus

Point set topology. Functions of several variables. Differentiability of functions between normed vector spaces. Series, Power series, Fourier series. Riemann-Stieltjes integral (functions of one variable). Riemanns integral (functions of several variables). Analytic functions.

Real Functions

Lebesgue integration. Abstract integration. Differentiation of set functions. Fourier integrals.

Algebra

Group theory (decomposition theorem for finitely generated abelian groups). Rings and fields. Construction of the fields of rational numbers and of real numbers. Elements of Galois theory

Linear Algebra and Geometry

Linear algebra. Linear transformations. Affine and Euclidean geometry. Determinants and volume. Foundation of geometry, non-Euclidean geometry. Differential geometry.

Functional Analysis

Hilbert spaces and Banach spaces. Spectral theorem for compact operators.

This program is intended to occupy approximately 50 percent of the study time in the first three years. The rest of the time in these years is spent on a subsidiary subject or on additional mathematical studies. Naturally the students have possibilities of following several other more or less advanced courses both during the first and during the second part of the study.

During the graduate study the student concentrates on a special subject, and inside this subject he chooses a narrow field where he (as far as possible) can get in contact with original research. Naturally, not all the students have the abilities or the scientific interest to do research. But in any case, it is an enrichment for the future teacher to get acquainted with fields where research is done.

The major advantage of having the same education for high school teachers and scientists is that the students can study mathematics in a purely academic atmosphere without being forced to choose their professional career too early. During the years at the university the students gradually develop their main interest, and very often they end by choosing another career than originally intended.

This study form has however the undeniable shortcoming in that it does not help the intending teachers to utilize their scientific knowledge in their high school teaching. Experience has shown that such help is often much needed. For that reason we have tried at one university to arrange seminars on elementary mathematics from an advanced standpoint. Our experiences are still too new to draw conclusions. But the seminars have been met with great interest both from students with primarily scientific and from students with primarily pedagogical interest in mathematics. This is the most positive result from our seminars. It is very important for the development of mathematics in high school that the teachers are genuinely interested in mathematics, but it is just as important that the scientists take a serious interest in the problems of the high school.

A TURKISH EFFORT TO IMPROVE SECONDARY SCHOOL MATHEMATICS AND SCIENCE

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Introduction

In the second half of the twentieth century, the importance of science and technology to every nation -- emerging, developing, or developed -- does not need to be defended; it is apparent to the most casual observer. The explosive expansion of fundamental scientific knowledge that began in the first half of the century is a potential source of drastic improvements in a nation's economy, capable of bringing to every citizen more food and water, better clothing and shelter, greater opportunities for education and wage earning, and in general a healthier, safer, and more rewarding life. But if a nation is to be able to exploit modern science and technology, it must have men and women competent to translate the resulting knowledge into means for improvement of the national economy.

Turkey has demonstrated its ability to produce first-rate fundamental scientists. Yet many of these outstanding scientists have expressed concern over the question of their successors. They report that the best scientific talent coming to the universities from the schools is seeking careers in applied sciences such as medicine and engineering, where the financial rewards are clear, and that the basic physical, biological, and mathematical sciences are attracting only the second best. While this may be an over-simplification, there is genuine concern among scientists that far too few capable students are entering university teaching and research in the basic sciences to meet even the current demands of Turkey -- demands that are bound to grow with Turkey's expanding industrial economy.

Of equally great concern to Turkish scientists is the fact that year by year fewer university graduates in science and mathematics are choosing high-school (lise) teaching as a career. They note with alarm that at a time when the school population is growing more and more rapidly, proportionally fewer teachers are in a position to make clear to their students the rewards -- intellectual as well as material -- available to them in careers in research and teaching in the basic sciences.

Finally, there is the question of human talent as a national resource. Turkey has long had a reputation for paying special attention to talented youth in the performing arts -- music, ballet, theater, etc. In an age of science and technology, argue the scientists, why should Turkey not pay special attention as well to the scientifically talented on whom the scientific and technological progress of the nation must depend in the next decades?

An ideal solution to these problems is easy to imagine: take all teachers of science and mathematics now teaching in all of Turkey's high schools, bring them abreast of the most recent developments in mathematics and science, and provide them with all the textbooks and all the laboratory equipment they need. In this way more students would be attracted to high-school science teaching, scientifically talented students would be given every opportunity to develop their talents, and the universities and industry would be assured of an adequate flow of competent young people into research and teaching in the basic sciences. But this solution is hardly realistic. No nation in the world has the financial and professional resources -- the money and the scientists -- required for the job.

Launching the Project

For a realistic solution, the scientists of Turkey arrived at what has come to be known as the National Science Lise Project. The germ of the idea was this: Bring some of Turkey's best high school science teachers together with some of her most dedicated scientists, let them explore together modern developments in classroom and laboratory materials and methods with a view to producing modern high school programs, use these programs first in a special high school for Turkey's best scientific talent, and later adapt them for use in regular high schools throughout Turkey.

To launch the Science Lise Project, the Minister of Education in April 1962 appointed a study committee of five, two scientists, two members of the Ministry, and a fifth member grounded in science and experienced in school management and teacher training. This committee submitted a report that was promptly accepted by the Minister of Education.

To oversee the details of planning and conduct of the project the Minister of Education appointed an administrative and advisory committee consisting of four scientists and three members of the Ministry. It is significant that here, as in the case of the founding committee, the Minister looked to the scientists of Turkey for guidance in an undertaking of great importance to the development of the nation's education in the sciences.

To house the Science Lise Project, a specially designed coeducational boarding school for 300 students was constructed on a thirty-acre tract of land overlooking Ankara, and its laboratories and classrooms were outfitted with the most modern equipment and furnishings.

Each year students from the graduating classes of the middle schools (junior high schools) of Turkey are selected by national competitive examinations for admission to the first of the three years of the Science Lise. Over 10,000 applicants are screened by a first examination, testing general achievement; the top 1,000 of these take a second examination, testing scientific potential; and 100 of these are finally selected for admission.

Training the Teachers

While the buildings were under construction, thirty high school teachers of mathematics and science were selected by national competition. They were assigned half-time teaching in Ankara high schools in order to devote the rest of their time to a three-year program of in-service training and course writing in cooperation with a group of university scientists. Less than half of them were engaged in teaching in the Science Lise; the others worked in other phases of the project. The teachers of other major subjects were also selected by competition, and some of them -- notably the foreign language teachers -- were given special training.

The Turkish scientists participating in the intensive teacher-training and course-writing aspects of the project are from the science faculties of Ankara University and Middle East Technical University. Some fifteen have been involved, including the Turkish co-director of the project.

American scientists and science teachers have also cooperated in the project, on the assumption that the wealth of American experience in teacher training and curricular change could be exploited with profit. Florida State University has provided the American co-director and other consultants, and managed those parts of the project based in the United States. Four American teachers came to Turkey to work with the project during 1964-1966.

In the summer of 1963 the Turkish professors visited the United States to examine the major American teacher-training and course-writing projects in detail. During 1963-64 they conducted in-service programs in their specialties for the benefit of the thirty teachers selected for the project. The entire group -- professors and teachers -- then spent the summer of 1964 at Florida State University, working on curriculum and teaching materials for the first year of the Science Lise. There they conferred frequently with American scientists and teachers, observed some of the new science programs in action in the public schools of Florida, and visited the Bronx High School of Science in New York. Following their return to Turkey and the opening of the Science Lise in October 1964, the program of in-service training and course writing was resumed. At a summer workshop in 1965, the first-year materials were revised in the light of experience and the second-year materials produced. This process was repeated in 1966, and in 1967 the materials will receive a final polishing.

Project Extension

Naturally, the long-range aspects of the Science Lise Project go far beyond the establishment of the Science Lise and the production for its students of new programs in biology, chemistry, mathematics, and physics. After several years of working together, the project professors and teachers have boiled down, so to speak, to a compact and well-coordinated team which is in a position to make contributions of the greatest importance to science education generally in Turkey.

In the summers of both 1965 and 1966, five-week summer institutes were conducted by the project at the Science Lise. More than 200 teachers from all parts of Turkey were not only introduced to the new materials and methods at these institutes, but sent back to their schools with substantial kits of equipment and supplies to enable them to put into action what they had learned. Participation by professors from other Turkish universities served to widen the nation's acquaintance with the project.

Again, in the spring of 1966 the project mathematics teachers conducted a Saturday conference on the teaching of modern mathematics for teachers from Ankara Lisos. It was a modest but successful beginning. A few weeks later the project sponsored a more elaborate undertaking -- a five-day international conference on mathematics education, attended by over fifty Turkish mathematics professors and teachers from all parts of Turkey, by numerous Ministry of Education officials, and by several European and American mathematicians and mathematics educators.

These activities have made the participating teachers and professors restless for change. One early result was a request for the project materials at one of Anakra's ordinary lises, and they are being used there this year under supervision of project teachers. Similar opportunities are bound to follow, especially if the Ministry of Education acts on a current proposal to incorporate the project in an expanded central agency for science curriculum development, properly organized and placed in the Ministry, and with appropriate power and budget. The problem now is to induce carefully planned and controlled spread of the new materials and methods, lest failures undercut the progress made to date.

The Science Lise Project is an example of cooperation by many agencies. The Government of Turkey, through the Ministry of Education, provided funds for the construction of the Science Lise and is financing the usual costs of operation. The Ford Foundation has provided foreign exchange required for the program teacher education and course writing, and for important equipment and furnishings. Middle East Technical University provided land for the campus and managed the building construction. Ankara University provided space and facilities for the project offices until the Science Lise opened. And as noted above, professors from Middle East Technical University, Ankara University, and Florida State University have worked together with Turkish and American teachers in the development of the new programs and teaching materials.

Everyone concerned with the Science Lise Project hopes that this pioneering effort to improve secondary school mathematics and science in Turkey will be helpful in pointing the way for other nations faced with similar problems at comparable stages in their development.

THE STATE OF REFORM OF MATHEMATICS INSTRUCTION IN BELGIUM, 1966

G. Papy, Universite de Bruxelles

The intensive and extensive experiment of reformation of mathematics instruction in Belgium enters this school year into its ninth year. The experiment flows to the terminal class of the scientific section of secondary instruction (17 to 18 year olds) it is possible and useful to give an account.

In this movement, there can be distinguished three main periods, each three years in length.

a) 1958-1961: Experimentation with the Lenger-Servais program in certain Normal School classes, those in the Nursery School section (future teachers of children 3 to 6 years of age).

b) 1961-1964: Transferring the experiment to instruction into the first cycle of general secondary school instruction, pupils ages 12 to 15 years. Generalizations of modern programs in hundreds of classes since 1964. In 1961 there was an attempt to modernize the instruction of a class of pupils of age 15 years who had not received any modern instruction in the first cycle (12-15 years of age).

c) 1964-1967: The first experimentation of modern instruction in a class of students of the scientific section (second cycle of secondary instruction, 15-18 year olds) who had received modern instruction from ages 12 to 15 years. Since 1966 there has been a repetition of this experiment in a new class - taking into account the first experimentation from 1964-1967.

a. The First Period, 1958-1961

The Lenger-Servais contained subject matter basic to modern instruction which was first proposed about 1950. It was certainly influenced by the work of the International Commission for the Improvement of Mathematical Instruction and by the work of Northrop and others at the University of Chicago. Since that time, it has appeared wise to promote experimentation in Belgium. Also as little originality as possible was exercised in the choice of subject matter. The instruction was limited to those concepts and general results recognized as important by the majority of experts in the field.

The Lenger-Servais program took into account the specific goals of instruction in mathematics of the future teachers of children 3 to 6 years of age. It was for this reason that it made use of particular kinds of ideas - those of sets, relations, and topology. The authors did not intend to recommend, even in an indirect way, how sets, relations, and topological ideas should be taught to three to six year olds. Nevertheless, it seemed appropriate to them that the future teachers should be informed of the nature of mathematical concepts basic to our times in a manner favoring certain approaches to this spirit in the children's games. It also appeared to them that Venn diagrams and certain very elementary ideas of topology seemed somewhat like spontaneous designs of children, often so beautiful and so mysteriously interesting. This experiment permitted certain novel conjectures to arise which appeared to be interesting for further study.

The students who choose the section "Maternelle Normal" are motivated to occupy themselves with the education of young children. Generally these students have attained very weak results in mathematical study in their previous classes. Certain of these students display nothing but their basic animosity toward mathematics and the professors who teach it. It would be interesting to see what would happen if these students of weak receptivity would be brought into contact with new ideas about mathematics at a level of prime importance.

The mathematical ideas appearing in the Lenger-Servais program were very elementary. Nevertheless, it became quickly apparent to the promoters that the pedagogical sequence of the subject raised problems in relation to the mathematical sequence. So, they decided to assure themselves of the collaboration of a purely technical professional mathematician. This university professor, besides being very skeptical, appearing even hostile, with respect to the tentative undertaking, accepted nonetheless to lend himself in good grace in the role of a subaltern. Because of the promising results that were obtained, this mathematician became, without a doubt, the first opponent to be converted by the evidence of achievement. After the second year of the experiment - under jeers from certain of his colleagues - he decided to take over an experimental class by himself.

At the start of the experiment, Venn diagrams had been used with promise of success. Following tradition, one employed at the start a system of colored surfaces or hatched surfaces. It was in actual contact with the pupils that the method of using colored strings was substituted. The hatching - according to a suggestion from one of the pupils - was used to indicate the empty regions. Spontaneously, these pupils, having been placed in a favorable pedagogical situation, discovered a process, proposed by Venn himself. Activity instruction and the inspiring contact with pupils made it possible for the mathematician to discover, in 1959, the pedagogical system of multi-colored graphs. Throughout this first experiment there was revealed an intuitive support and a precious ideogram for more advanced instruction.

Likewise, from this first experience, there appeared a germ of a method of introducing real numbers through the systematic use of positional numeration (especially, for pedagogical reasons, the base two). Starting with the Mobius band, introduced in an intuitive manner, one could proceed, little by little, to the elements of general topology. We started with very simple notions: neighborhoods (the prisons of continuity), open circles with the convention suggested by the pupils that green-red be used to distinguish respectively open and closed circular regions.

This first experience soon extended to numerous classes, was extremely fruitful. It produced new pedagogical procedures that permitted a glimpse of a restructuring in a most far-reaching manner of the mathematics instruction at the elementary level. The instruction given in these classes was a pleasant climate. The hostility of the pupils against mathematics had completely disappeared. One perceived that the children of today could be tuned in with the mathematics of today.

Parallel to this experimentation, consent had been given to a great effort to diffuse the concepts of modern mathematics and a pedagogy of its instruction to in-service teachers and students in higher normal schools. The following teaching conferences were held:

Arlon 1, 1959: A presentation of the first results of the experiments and the study of the brochure, Arlon 1, on sets and general topology. Demonstration lessons were given to pupils in the new classes.

Arlon 2, 1960: A presentation of a course (cyclo style) was presented for use in modern classes. (First elements of Modern Mathematics). The use of graphic methods was communicated for the first time.

Arlon 3, 1961: The first initiation to the theory of groups with the aid of favorable pedagogical devices.

In the course of this first phase, the director of a large secondary school in Brussells, Mr. Oscar Guillaume, founded a free mathematics club open to all those students well-disposed toward secondary instruction. This club was used mainly as an enterprise of reform for it permitted, without any constraints, whatsoever, the carrying out of numerous experiments on precise topics of modern mathematics. The club was an enormous success. The participants, all volunteers, showed great aptitude for mathematics. In a short period of time, it was possible to make precious experiments.

b. The Second Phase of Experimentation, 1961-1964

The mathematician who played a subalternate role in the debut of the first phase seemed sufficiently converted and without too great expectations, he wrote himself a "Suggestions for a new mathematics program for classes of 12-year olds." Because of the good results of the conference Arlon 3, and thanks to the determination expressed by Mr. Henri Levarlet, then the General Director of Secondary Education in Belgium, the Minister of Educatio decided in favor of trying the proposed program in the sixth class (12-year olds). Since that time the experiment has progressed regularly, without any interruption, attaining each year a new higher class in the echelon of instruction.

The new program thus gained an extension from the classes for 12-year olds up to those for 15-year olds. The experimental program was optional in the schools but acutally hundreds of classes each year have adopted the new program. Finally, in 1965, it was decided that the programs proposed by the Belgium Center of Mathematical Pedagogy, created during the period 1961-1964, will be the only ones permitted for experimentation, and further, that the modern program will be obligatory in all classes for 12-year olds in State instruction beginning in 1968.

Several words are in order concerning the Belgian Center of Mathematical Pedagogy, now actually known as the basic collective Center of Scientific Research, by ministerial initiation. The research in mathematical pedagogy is carried out by the promoters, a mathematician and an experimental educator, with the aid of several assistants. In December 1966, these assistants numbered 10 (4 Belgians, 2 Argentines, 1 Brazilian, 1 Canadian, 1 Greek and 1 Turk). The Center organizes, without cost, a course for instructors in 25 cities (20 afternoons per year). Three thousand instructors of Belgium have actually taken the course, without any expense to the Belgian Government. The courses are given free. The teachers themselves pay their travel expenses and printing of the syllabi. A great current of aposotlic feeling animates the Belgian teachers of modern mathematics.

The first program, mathematics for classes of 12-year olds, had a single author. The center very early created a Commission of Programs. This Commission included university professors, inspectors, heads of schools and teachers of experimental classes. The programs were established by the Commission and placed into professional teaching, taking account of the experiences in the course. The Center assured the publication of the textbooks in view of the experimental nature and the recording of the class teaching. The Center also organizes sequences of instruction at the request of teachers, using a cycle of courses. These courses benefit from the financial support of the Ministry of National Education.

We now examine briefly the content of the programs for 12 to 15 year olds, as well as some of the methods of unfolding the instruction. It must be made clear, that this instruction has been given to classes that are in the common branch of secondary education. Mathematics is taught four times weekly, each period

45 minutes in length. The content of the program conforms to recommendations, views, and motions expressed in the several reports of a number of international converences of groups of experts in mathematical education, as well as pure and applied mathematicians: Royaumont, Dubrovnik, Aarhus, Budapest, Athens, Frascatti, Echternach. These recommendations, to be sure, are limited to the proposals of several large view-points, without concern for the organization of the material into course structure, subdivided by years of study. It remained to effect a reconstruction of elementary mathematics, necessary and preliminary to designing the entire program.

The program outlined in the following is schematic.

Age 12 years: Sets, relations; the ring of integral rationals; the introduction to affine geometry. We limite the discussion to remarks concerning the teaching of geometry. Certain persons have requested that in the reformed instruction some place be reserved for geometry. Our experience has confirmed the fundamental role of the euclidean plane. However, it must be clarified in a new way. It is reconstructed by presenting the fundamental structures of mathematics. As a support of geometry, and more fundamental than geometry, the strucutres remain important in the entire sequence of mathematics study. This development of geometry conserves for it an intuitive, familiar, and motivational character of study.

The antagonism of rigor versus intuition appears to have ended. It is only since 1900 A.D., with Hilberts' Foundations, that we have had at our disposition a rigorous development of geometry. Previous to that time, it was inevitable that instruction in geometry had recourse to intuition that gave a nebulous character to geometric reasoning. The Foundations of Hilbert is not in any manner a textbook for secondary instruction. A half century of progress in mathematics has placed in evidence those structures which render a rigorous exposition of geometry simpler and more intelligible. But, we add, the study of the vectorial euclidean plane is based on a complicated structure. - How can we introduce it to the students in a manner to interest them in the structure? Well, this poses a pedagogical problem.

The metric plane is a complicated structure, It is no longer adequate as an introduction to mathematical reasoning. One begins, then, with an underlying structure, logically more simple. The instruction of sets facilitates this introduction. For very good reasons we must begin by stating clearly what is accepted. This is the approach which one finds in the mathematizing phase of problems of applied mathematics

State clearly that which is accepted

Do not say everything at one time

State certain accepted things, little by little.

This is the progressive axiomatic approach.

The useful axiomatic method is that of the physicist. State that which is accepted as idealized from a real situation and say it little by little. The first axioms of geometry are very simple. They are introduced carefully in a slow axiomatic approach which well clarifies the difference between points and dots which represent them. But always reason with large dots as points. The microscopic drawing escapes an important problem; it does not solve it. After stating the usual axioms of incidence, one finds himself in a simple logical situation, expressed clearly by the language of sets.

How is reasoning accomplished on the basis of traditional figures where one sees the response? Venn diagrams give the problems the aspect of a Roman guard. In representation by the traditional figures, the lines and planes are given a "set" form. To resolve a problem it is necessary to reason. We do not abandon the use of traditional figures. We double the effect of intuition by the intervention of Venn diagrams, supported intuitively by the logical structure of the situation. Yet it is necessary to guard against a too great slackening of reasoning with sets in its formalistic aspect. The Venn diagrams permit a rapid approach to synthetic reasoning.

The set point-of-view is absolutely indispensible in the study of elements of general topology. The traditional introduction of the course in analysis is the least revolutionary. For example a bristol-board does not permit us to make the indispensible distinction between an open and closed square. But the useful intervention of the convention red-green calls up the distinction to a high degree. Because of the lack of space, for greater detail on the organization of the course in geometry, the reader is referred to Mathematiques Moderne, books one and 6.

Thus far, there has been a lack of proof involving several steps, so necessary for the eventual constructions. At the most, we have had a preview of "one-step," in the sense of the mathematician, not the logician. A demonstration with several steps always necessitates a certain effort. Why do we consent to make such an effort if it is only to prove that which is a-priori not certain?

There is a fundamental pedagogical error in the traditional programs. Frequently 12 and 13-year olds study intuitive geometry. Without doing any mathematics one begins to see correctly a number of things, most frequently stripped of all ulterior interest. Sometimes one goes as far as helicoid surfaces. Then, at age 13 years, one brutally changes the attitude and begins a study of a new discipline - demonstrative geometry. One abandons the method of contemplation and begins to crack the skull with demonstration. To prove what? That at a point on a line one can erect one and only one perpendicular to the line - that which the children have known for a long time. Poor children who do not legitimately rise up in insurrection! Why crack one's head to prove that which is already known? They capitulate under the demoralizing action of an extinguisher! In truth it is well-known that the demonstration in question is bird-catching. The presentation to children as demonstration is a veritable deception, of the nature of giving the children a dangerous false idea, that is indeed a true demonstration. In this material the traditional programs teach nothing!

They give a false proof of a known fact.

They teach neither the fact, nor that which constitutes a proof.

It is necessary to choose carefully the goal of the first demonstrations. The problem of the parallel projection of equipollent ordered pairs is a good situation for introducing proof. Are the parallel projections of equipollent couples (ordered pairs also equipollent? The class is divided in its response! How shall we begin? How shall we decide? Certainly not by vote! By an argument - by means of conviction - by a demonstration!

At each step, one passes from one piece of information to another which clearly follows. The pedagogical procedure of demonstration by film strips furnishes an intuitive support to the demonstration itself. This is first presented in a non-verbal form. Certain children, incapable of understanding a verbal demonstration appear to follow a demonstration by strips of drawings. This pedagogical means aids children in attacking a demonstration. By the third demonstration they themselves propose that they be permitted to discover the proof.

After the first presentation of a proof one next establishes a sound track film strip - using a verbal justification of the passage from one image to the one following. Finally, the third step consists in learning to establish the "text" of the proof in the usual vehicular of mathematical language.

<u>13-Year Olds: The Class of Computation</u>

In this year there are introduced simultaneously and progressiv ly the two essential fundamental structures: the ordered field of real numbers and the real vectorial plane. One starts with the additive group of vectors. Its model - the points of a plane - is a fascinating and fundamental pedagogical situation. There are also a sufficient number of examples to sketch a small complete theory of a group. The exercises must be selected to convince the student that the possession of the concept will make them strong and permit them to solve problems which previously caused discouragement.

The real numbers are introduced by means of positional numeration. The binary system permits one to present the iterative process of sub-graduation of the line. The theorem of Thales is a spectacular application of this method. It also allows the introduction of homotheties. Translations and homotheties introduce the addition and multiplication of real numbers by using the important procedure of transferring a structure by bijection (isomorphism).

The presentation of the real vectorial plane is the culminating point of the year's work. The course ends by putting into operation the useful machine which demonstrates the theorems of geometry. In the structure of the real field there is introduced the theory of computation built solidly from fractions to real numbers. For further details, see Mathematiques Moderne, 2..

Age 14-Years: Equations of the line are established in the structure of the vectorplane. The ring of functions R --> R and the sub-ring of polynomial functions and the solution of systems of equations by the Gaussian method, including the effective calculation of the general solution of a system complete the year's study.

A main point of the program for this year is the geometric metric plane introduced by orthogonal symmetries. By virtue of formation one starts with central symmetries which one controls by the pedagogical means of numbered points. The scaled lines introduce the group of isometries and sub group of displacements, rotations, and translations. The study ends in the introduction of the scalar product and the euclidean vector plane. The scalar product is treated rigorously but always following the reasoning with figures in a manner that does not prevent recourse to geometric intuition.

The year's study ends by placing into operation the machine tool, the euclidean vector plane, to establish an industrial manner, the theorem of Pythagoras, the Cauchy-Schwartz inequality, and their consequences. Angles are introduced as rotations that have lost their centers, that is to say, as a transformation by a translation. For complementary information, see Mathematiques Moderne, 2 and 3.

Parallel to the experiences in the classes for 12 to 15 years old, the teachers are given information in the reports

Arlon 4: Vector spaces

Arlon 5: Exterior algebra

Arlon 6: Analysis (Now being revised in view of the experimental teaching of analysis.)

Perhaps it is convenient at this point to mention a new sign of skeptical wisdom of one of the promoters of reform in Belgium. He asks: Is it necessary to give so much effort to the organization of a modern curriculum for 12 to 15 year olds? Is it not sufficient to begin with 15 year olds in introducing modern ideas, and to regard the traditional instruction for 12 to 15 year olds as a preparation more or less intuitive?

The answer: Experience has shown that to retard the introduction of the ideas of sets and relations bottles up the course of the higher classes. Moreover, traditionally bad habits having been grasped, an important part of higher instruction consists not of supplementing what was not learned, but in making students forget what they had learned. What a loss of energy! A posteriori, we have established the fact, that in the class of 15 year olds who have had modern instruction from ages 12 to 15 years, one can go very much further and avoid for the students the sorrowful moments of reconditioning.

Third Phase, 1964-1967

We describe the instruction in the scientific course for students who have had a modern course in the period 12 to 15 years of age. The course for this section has seven weekly periods of instruction, each 45 minutes. The limitation to the scientific class was made because it was not possible to conduct an experiment simultaneously in several classes. We were well aware of the problems posed by instruction in mathematics to non-scientific classes, ages 15 to 18 years. The first experiment was in a terminal class of 17-year olds in the scientific section. Taking into account all the collected results of past instruction, a new experiment is to be broached in the classes of 15 year olds, to be followed in subsequent courses in the years 1967-1969.

We shall be more brief in describing the modern program for the scientific section (15-18 years). In view of our second repeated experiment we present the program in an amended form for which the changes from the original are minor.

15 year olds (Here we have a psychological turn-over! As a convenient situation to the Student's morale). The structure of the real field and of the euclidean vector plane have been attaine after a long ascension, starting with axioms, easily accepted, of an intuitive character, or "evident." This time one takes the system of axioms (d∈ R, +, ·, $\leq$) and the euclidean vector plane as a point of departure. This return is not to be a pretext of a phenomenon of regression which consists of proving that which one already knows. The new point of view is to be considered a base of departure for new conquests.

In the entire study of vectors, the theorem of the basis of Grassmann (Steinitz) is fundamental. The results of application for the case of vectors of 2 or 3 dimensions are ponderous and dull. The pupils have already met a large number of examples of vectors (vectors of polynomials, vectors of equations, etc.). The discovery of a good pedagogical procedure which permits the proof of Grassmann's theorem at an elementary level is one of the most important results of our experiment. The following sequenc is suggestive:

Vectors in 2 dimensions

Ring of linear transformations

Linear transformations defined by the image of a basis

A surging of interest by the use of multi-colored graphs

Coordinates, 2 x 2 matrices

Orthogonal transformations, Orthonormal bases

Orthogonal matrices - self practicing theory and calculation with matrices

Symmetries, Rotations

Similitudes - direct similitudes

The field of direct similitudes

Half turns. The two-fourths of a turn

Orientation. The fourth of a turn, i.

The complex field

Trigonometry

Combinatorics

Factorization in $(Z,+,\cdot)$

For further details, consult <u>Mathematiques Moderne</u>, 5 and 6.

<u>16-Year Olds</u>

Analysis and Topological Spaces

$\pi\ \mathcal{T}_{\mu s}$; D, $\mathcal{T}_{\mu s}$; R, $\mathcal{T}_{\mu s}$; $\overline{R}\ \mathcal{T}_{\mu s}$; $\bar{\omega}\ \mathcal{T}_{\mu s}$

Topology of metric spaces

E, $\mathcal{T}_{\mu s}$ (The usual space of three dimensions)

R^2, $\mathcal{T}_{\mu s}$ Topology. Homeomorphisms. Neighborhoods; Adherence, Product spaces.

Continuity, limits, sequences, derivation, integration.

For more details, consult the teacher references, Arlon 6 and 8.

Vector Spaces

The theorem of Grassmann

$\dim A + \dim B = \dim [A \cup B] + \dim [A \cap B]$

$\forall f \in a^{p}(V - W): \dim fV + \dim f^{-1}\{0\} = \dim V$

(See F 2, Initiation into Vector Spaces)

A new view of problems in systems of linear equations

Matrices. Determinants. Affine Geometry of usual space starting with a vector space of 3 dimensions.

Arithmetic

The ring of residue classes $(Z_n, +, \cdot)$

The ring of polynomial functions over a ring

Rings (See Mathematiques Moderne, 5.)

17 Year Olds. This is a year of calculus dedicated mainly to intending engineers and to preparatory engineering schools.

Proof of Taylor's formula

The integral as a limit of sums

Logarithms and exponents. The number e

Length of certain curves. The number π

Measure of angles

The group of angles $(A, +)$ and isomorphism to $(R_{/Z}, +)$

Circular functions $R \to R$

Differential Equations

The final form of the theory of linear equations

Bilinear forms. Conics

Euclidean vector space of 3 dimensions.

This instruction of analysis has been made possible only by the previous modern preparation. Difficulties do not arise. The clarity and the simplicity of the exposition results in its own rigor. Parallel to the experimental teaching, teachers continue to obtain their needed information from

Arlon 7: On the scalar product of vectors

Arlon 8: Lessons in Analysis effectively carried out by Frederique in classes for 16 year olds.

preparation is Arlon 9 on integrals

Problems of Tomorrow

a) The introduction of elements of probability into the program. We are working on this in collaboration with Professor Jean Teghem.

b) A systematic study of mathematical reasoning which intervenes in the new instruction of elementary mathematics.

c) Exploiting to advantage audio-visual procedures.

The entire movement has been followed with great enthusiasm. On the first of December, 1966, a large conference for giving information on the status of reform, with the title Reform In Progress brought together 1,700 Belgium teachers of secondary mathematics. The principal tasks of the future are the reform of instruction in primary school mathematics and to prepare, as soon as possible for the second reform in secondary school instruction in mathematics. There is no halt. Let us go forward!!

References

Mathematique Moderne 1. Sets, Relations
2. Real numbers, vector plane
3. Here is Euclid
5. Arithmetique
6. Plane geometry - euclidean vector plane

Published by Didier, French; Macmillan, English; Klett, German; Eudeba, Spanish; Didier, Flemish.

Arlon 1 Elements of Topology
Arlon 6 Document for Teaching Analysis
Arlon 7 Document on Vector Spaces and Scalar Product
Arlon 8 The Course in Analysis by Frederique for 16-year olds

Published by Centre belge de Pedagogie de la Mathematiques, 183 Avenue Brugmann, Bruxelles 6, Belgium.

F(1) Affine Plane Geometry and Real Numbers
F(2) Introduction to Vector Spaces

Published by Gauthier-Villars (French).

THE ANALYSIS CURRICULUM

Andre Revuz

Basic Rudiments of Analysis

Continuity and limits are known as difficult and as teachable only to students of sufficient intellectual maturity. The majority of elementary courses do not seek to go beyond vague explanations, insisting more on technical aspects of derivation, for example, than on profound significance. The latter is studied in higher-level courses.

The basis for this attitude lies more in prejudice than in the results of experience. Experience, shows that intellectual maturity is much less related to the age of the student than to the education he has had; also that true mathematics concepts are well received by students provided they are presented in all their clarity and with careful motivation. If the concept is difficult, the solution does not lie in delaying its presentation; rather on the contrary in preparing it much earlier. The motto of mathematics instruction should be "early and progressively."

With respect to analysis, the basic rudiments that are encountered at every step and which must be introduced from the beginning are <u>linearity</u> and <u>approximation</u>.

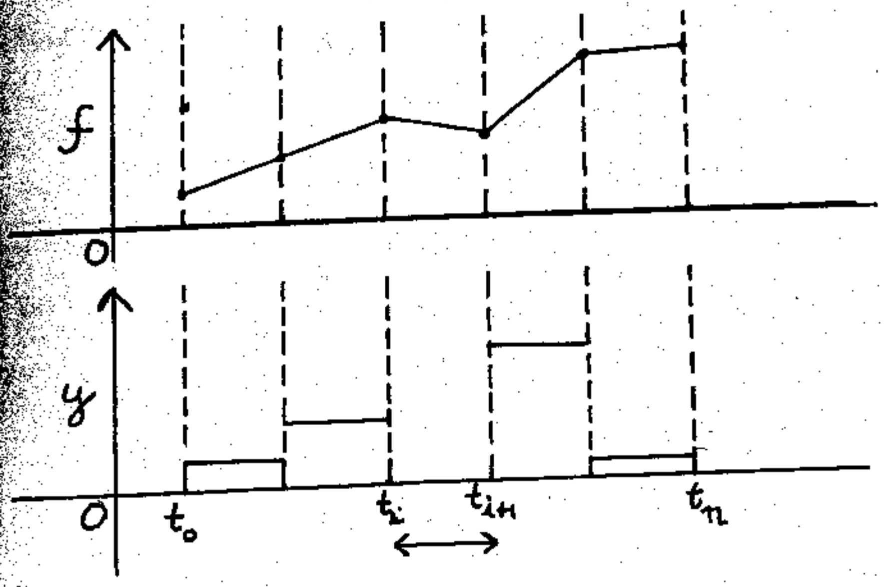
The sequence of instruction in Analysis from the beginning of the secondary level to the early years of the University level, I believe, should be the following:

I. Linear interpolation. Piece-wise linear functions and associated step functions.

These questions may be presented profitably to students 10 to 12 years of age. At this stage the intention is to use the study of concrete situations as preparation for introducing the basic rudiments of analysis and already providing some simple tools which can be correctly manipulated.

The students at this age know the linear function $x \longrightarrow kx$ which is presented to them really by way of the complicated "proportional magnitudes," while the sets that describe x and kx are not made very clear. The student knows that these are "numbers"; often he knows no more but is satisfied with this for the beginning. Similarly he may be satisfied with the experimental comparison of finding that on coordinated paper all points with the coordinates x and kx are alined. This being the case, it will be seen that in practice correspondance between magnitudes is never known except by a finite number of values (experimental results, tables of numerical values); but it may be interesting to extend the correspondance to other values as far as is possible with simplicity, to effect an interpolation. The simplest of these interpolations is linear interpolation: the reason for this simplicity lies in the fact that the application $x \longrightarrow kx$ is an endomorphism of the additive structure of Z (k must then be a whole number), of Q (k rational) or of R. (In this way we also obtain the only endomorphisms for Z and Q and the only endomorphisms of R which are also monotonic or continuous. These are also isomorphisms for R and Q.).

Linear interpolation replaces the correspondence in which only a finite number of pairs is known with a piece-wise linear function (which will hereafter be designated by p.l.f.). For example, this is the case with graphic representations of railroads. The slope of each of the segments of the path of the graph of a train is of interest: in the case of a train it is the commercial speed over the corresponding run. Another graph could be traced made up of segments parallel to the axis of abscissas and using the value of the commercial speed in the interval containing the point for respective ordinates.



This gives us an ordered couple form of one (p.l.f., f) and a step function ϕ joined by the following relations:

If $t \in]t_i, t_{i+1}[$; $\phi(t) = \dfrac{f(t') - f(t)}{t' - t}$ for $t' \neq t$,

$t' \in]t_i, t_{i+1}[$.

For $t \in [t_i, t_{i+1}[$, $f(t) - f(t_o) = \sum\limits_{j < i} u_i(t_{j+1} - t_j) + u_i(t - t_i)$.

Here, then, we have simple cases made up of a function f and its derivatives ϕ (derivative except in a finite number of points. The values of ϕ and the points t_i are not given and have no practical interest.).

It may be easily seen that the sets of the p.l.f. and the sets of the step functions have a natural vector space structure over R and that the mapping D that makes ϕ correspond to f and the mapping I that makes f correspond to ϕ (establishing, for example, $f(t_o) = 0$) are linear and mutually reciprocal. Thus we have the beginnings of the theory of differentiation and the theory of integration. The important theorem of the mean is easily demonstrated in the present case:

if for $t \in]t_o, t_n[$ we have $|\phi(t)| \leq K$, it then follows

$$|f(t) - f(t_o)| \leq k|t - t_o|.$$

If the preceding study were made with students between 10 and 12 years of age, as I contend it should, we would not use the term vector space since under present conditions the majority of students do not understand the term. We can speak of basic linear properties of D and I. These at least can be easily shown, and we then have very important examples of vector spaces and linear applications for our study.

It is possible and interesting to give numerous examples of p.l.f. and associated step functions:

1) The table of a progressive income tax (step function) and the amount of tax found as a function of the income (p.l.f.).

c) A number of passengers carried between two stations by a common means of transportations (step function) and the number of "travelers-kilometer" handled by these means of transportation (p.l.f.).

d) The amount of electricity used in a department a function of time (step function) and electricity consumption (p.l.f.). Let us remember that much statistical data is tried out on "historical grounds" which are graphical representations of step functions with which p.l.f. can be combined.

All the precedent considerations, then, lead to the calculation of finite differences and to a variety of numerical calculations of obvious importance.

III. Introduction of the body of reals R

The best way to proceed here seems to be through axiomatic presentation. Actually the students already manipulate rational numbers and some irrationals ($\sqrt{a}$, π). The object, then, is essentially to make them aware of the properties they are using and to strike a complete and exact balance, contiguous with the structure of a totally ordered Archimedian commutative body and possessing the property of nested intervals. (For brevity's sake the foregoing terminology may be employed with students with an adequate number of examples of algebraic structures [simple ones] and ordered structures which will have already been seen. Actually this is only a minor problem. The main thing is, that whatever the vocabulary employed, the students should clearly see what can and can not be done with real numbers.)

Beginning at this stage it is essential to point out the equivalents of the property of nested intervals and the existence of an extending minimum for any extended set. This equivalence may be admitted without demonstration.

Along with the introduction of R, in the case of algebra, it is necessary to extract the notion of vector space starting numerous examples that the students should have learned by now: a) Many statistical data expressed by finite successions of numbers, successions that can be added and multiplied by a number. b) Examples of p.l.f. and step functions. c) The study (eventually purely experimental) of the transformations of a plane.

III. Progressive introduction of continuity, then the notion of limits.

In my opinion, it is necessary at this point to spend enough time to insure the establishment of the authentic mathematical definition of continuity.

I conceive of at least two approaches which are not mutually exclusive and obviously compatible:

1. Starting with the material studied in I (p.l.f. and step functions). If we use the word continuous in its intuitive sense no one will question that p.l.f.'s are continuous and the step functions are not. The problem is to give a mathematical definition that is unambiguous: we might start with the idea that a continuous function has no jumps; that is, that all jumps, no matter how small, must be excluded. In other words, regardless of how small

positive quantity may be, the function must vary less than that quantity when it approaches the value x_o at which we wish to exclude the possibility of jumps. We arrive at the traditional definition,

$$\forall \varepsilon \in R_+^*, \exists \eta \in R_+^* : |x - x_o| < \eta \Rightarrow |f(x) - f(x_o)| < \varepsilon$$

R_+^* designates the set of strictly positive reals. The use of the symbols $\forall$ and $\exists$ are very helpful in clarifying the ideas and giving impact to the statement.

2. Starting from the idea that every measurement can be approximated and a function that mathematically expresses a law of measurements must be able to guarantee that error in the value f(x) will be as small as is desired as long as the error in x will be sufficiently small. This brings us to the statement enunciated previously.

This can be done as in I by application of an interval of R to R but simultaneously by applications of a part of the plane within the plane and similarly of a metric space in another. It is a fact that the concept of distance is quite simple and easily accepted by students. The existence of various distances in the plane (linked to different norms of R^2) should be presented fairly early. This adds to intuition and shows that we can find an operator in the concept of distance when we divest it of its non-essential particular aspects.

The <u>notion of limits</u> is one of more delicate analysis and it should be introduced after continuity not before. One of the best ways of doing this, in my opinion, is to start from discontinuous functions that have limits (or a limit on the right and a limit on the left such as s functions, for example). The definition will be analogous to the one for continuity but one must insist on the fact that the value of the function at x_o, if there is one, does not enter into the definition of the limits at x_o; and the important thing here is the breaking up of the function into neighbouring values of x_o but different from x_o. For example, we find the definition in the case of a mapping f of a metric space E (provided with distance d) in a metric space F (provided with distance δ).

$$(\lim_{x_o} f = 1) \iff (\forall \varepsilon \in R_+^*, \exists \eta \in R_+^* : 0 < d(x,x_o) < \eta \implies \delta(f(x), f(x_o)) < \varepsilon)$$

Briefly explained, the property of the limits and the continuity which may be results from R being a topological body and R^2 (with its different equivalent norms) a topological vector space can be established at this stage simply and definitively.

IV. Differentiation of real functions of one real variable.

The point of departure is the approximation of such a function through a p.l.f. We know there are cases where this approximation is satisfactory in practical terms and where it is so good that the division of the interval of variation of the variable is more exact. It can also be noted that in practice it is useless to go beyond a certain minuteness of division (the ground cover by a train during $\frac{1}{1000}$ of a second is of no interest to any traveler; the production of a factory during $\frac{1}{10}$ of a second is insignificant, etc.). But the mathematician wants to go the end of the process; on the one hand to be able to satisfy any eventual need for greater precision, on the other hand for intellectual satisfaction and to achieve a situation which in the long run is conceptually simpler.

But it is clear that the indefinitive continuation of the partitioning will be significant particularly if the slope of the interpolation lines between x_o and x_o+h were x_o is fixed and h variable, varies less and less as h decreases. This leads to the notion of the derivable function for which we have $\lim_{h \to 0} \frac{f(x_o+h) - f(x_o)}{h} = l$. But if we wish to consider the number that this limit is, there is a significant fact about the precedent situation where a linear application was involved. It is possible to recover the linear application. We note that if we set $f(x_o+h) - f(x_o) = k(h)$ we have $\lim_{h \to 0} \frac{k(h)}{h} = l$ and $k(h) = lh + h \cdot \alpha(h)$, with $\lim_{h \to 0} \alpha(h) = 0$. This enables us to explain the differentiability of f at x_o by the fact that it can be written for h being quite small

$$f(x_o + h) - f(x_o) = l \cdot h + \alpha(h) \cdot h \quad \text{with} \lim_{h \to 0} \alpha(h) = 0.$$

The linear mapping $h \longrightarrow lh$ of R in R qualifies as linear tangent mapping of x_o to f; also as a differential of f in x_o.

Note that this mapping may be considered as the limit of the linear mapping $u \longrightarrow k(h)u$ when h tends toward 0, if the distance of two linear mappings of R in R is defined as the absolute value of the difference of its coefficients. (The set $\mathcal{L}(R,R)$ of linear mappings of R in R is isomorphic in R, and the precedent distance is coupled to the natural norm on (R,R).)

We shall call $D[f,x_o,\cdot]$ the differential of f in x_o and $D[f,x_o,h]$ its value in h. Then we have:

$$f(x_o+h) - f(x_o) = D[f,x_o,h] + \alpha(h)h \quad \text{with}$$
$$D[f,x_o,\cdot] \in \mathcal{L}(R,R) \text{ and } \lim_{h\to 0} \alpha(h) = 0.$$

The set Dx_o of the differentiable functions in x_o immediately becomes a vector space and the application $f \longrightarrow D[f,x_o,\cdot]$ of D_{x_o} in $\mathcal{L}(R,R)$ is linear. (The mapping of $D_{x_o} \times R$ in R defined by $(f,h) \longrightarrow D[f,x_o,h]$ is bilinear).

The two fundamental properties of the differential are the following:

1. If f: $(a,b) \longrightarrow R$ is differentiable in x_o, and if g: $(c,d) \longrightarrow R$, (with $f([a,b]) \subset [c,d]$), is differentiable in $f(x_o)$, the composite mapping gf is differentiable in x_o and its differential is the composite of the differentials of f and g

$$D(g \circ f,\ x_o,\ \cdot) = d(g,f(x_o),\cdot) \circ D(f,x_o,\cdot).$$

2. Theorem of the mean. If f is differentiable over [a,b] and if the norm of D[f,x] (that is, the absolute value of the derivative) is extended at every point $x \in]a,b[$ to the number M, then

$$|f(b) - f(x)| \leq M(b-a).$$

The proof uses the following facts: given ε there exists according to the hypothesis, a number η for every point $|h| < \eta(x)$ such that

$$|f(x+h) - f(x)| \leq (M + \varepsilon)\cdot|h| \qquad (1)$$

We then consider the set E of the points x, such that for any y

$y \in [a,x]$ we have $|f(y) - f(a)| \leq (M + \mathcal{E})|y - a|$ (2)

If E is not null, it is extended by b. It has an increasing minimum ξ (without this essential property of real numbers, the theorem would be false). The relation (1) is true for all $y \in \xi$. By continuity it is true for ξ. But by virtue of (2) it is true for $\xi + \eta(\xi)$, which proves that $\xi = b$ and that $b \in E$. With $\mathcal{E}$ arbitrary, the theorem works out.

The significance of the formulation presented here is

1) that it is valid for differentiable functions of an interval of R in any normal space;

2) the proof is much more natural than the proof of the theorem presented in the form of

$\exists c \in]a,b[$, such that $f(b) - f(a) = f'(c)(b-a)$

and that therefore the apparent accuracy is false since nothing can be said of c except that it belongs to the interval $]a,b[$.

Finally, let us point out that a proof very much like the preceding one could establish that if $D[f,x,\cdot]$ is an increasing mapping for every $x \in]a,b[$, then f is increasing on $[a,b]$. In addition, it is possible to deduce the theorem of the mean from this last result. The significance of the preceding exposition, where the structures involved are made clear, is that the results obtained can be generalized without difficulty.

3. Extension to the case of mappings of an open set of normed space E in a normed space F (in particular $E = R^p$, $F = R^q$.)

We need only see what has been useful for defining the differential to realize that the same results can be obtained in the following situation:

f is a mapping of an open Φ of a normed vector space E to a normed vector space F. Then f is said to be differentiable at the point $x_o \in \Phi$ if there exists an element $D[f,x_o,\cdot]$ of the space of the linear and continuous mapping of E and F such that for $||h||$ being quite small, we may write

$$f(x_o+h) - f(x_o) = D(f,x_o,h) + \alpha(h)\cdot||h||$$

where α is a mapping of the globe with center O of E in F, such that $\lim_{h\to 0} \alpha = 0$.

All the properties discovered in 3. for the differential and differentiable functions can be carried over with no difficulty. The theorem of the mean takes the form:

If the segment [a,b] is included in Φ, if for $x \in [a,b]$, we have $||D(f,x,\cdot)|| \leq M$, then

$$||f(b) - f(a)|| \leq M||b - a||$$

from which we deduce that if $||D[f,x,\cdot]||$ is bounded by M in all Φ, and if we designate $d_\Phi(a,b)$ the lower bound of the longitude of the polygonal paths which join a with b in Φ, then we have for any two points of Φ

$$||f(b) - f(a)|| \leq M\, d_\Phi(a,b).$$

Let us finally note that if $E = R^p$ and $F = R^q$, $D[f,x,\cdot]$ is a linear mapping of $R^p \longrightarrow R^q$ (and therefore continuous whatever norms are selected over R^p and R^q) and may be expressed in relation to the standard bases of R^p and R^q by a matrix of which the coefficients are the partial derivatives of the applications

$pr_i \cdot f$ where pr_i designates the i^{th} projection on R^q.

The essential observation to make here is that the step III to IV demands in the one hand the introduction of the idea of norm in a vector space which is very simple; the idea of the norm of the linear and continuous mapping of a normed vector space in another, and finally solid notions of linear algebra, which gives us another reason for introducing them.

It seems reasonable and possible to achieve a well thought program of secondary instruction which would permit the presentation of point IV in the latter part of the final year.

IV. Primitives - Integration

In the following we shall give only the most general directions of the development of the thought underscoring the motivation for each of the principal steps.

In the first stage the thinking may be limited to the consideration of mapping of an interval [a,b] of R in R, but at the end of the study-unit it will be apparent that all that has been said holds true for mappings of [a,b] in R^p and equally in any Banach space.

The definition of primitives offers no difficulty and the theorem of the mean guarantees that two primitives of the same function differ by a constant. Any derivation table read backwards gives a table of primitives.

But the first problem that may be posed is the following: if a derivative is only known approximately what can be said about its primitive? The theorem of the mean gives us the answer:

If for every $x \in [a,b]$, we have $|f'(x) - g'(x)| < \mathcal{E}$, then it can be stated that we also have $|(f(x)-f(a)) - (g(b)-g(a))| < \mathcal{E}|b-a|$, or even introducing the following norm which will be used in all that follows:

$$||\Phi|| = \sup \{[|\Phi(x)| : x \in [a,b] \}$$

and assuming that the condition $f(a) = g(a)$ obtains, we have the statement, if $||f' - g'|| < \mathcal{E}$ then $||f - g|| < \mathcal{E}(b-a)$. In other words, under the restriction $f(a) = g(a)$ the neighbouring derivatives have neighbouring primitives.

The reference to the approximation of a derivative f' by step function leads to an improvement of the theorem of the mean by demonstrating it in the case of a function that is not differentiable into a finite number of points; this is immediately clear or similarly into a denumerable infinitive of points (this is more involved). In what follows it will be supposed that "derived" means derived except possible in the points of a set that is at most denumerable.

The result obtained may then be expressed in the following fo
If the succession of derived functions (f'_n) converge, for the norm, having the derived function g', and if $f_n(a)$ converges towar the $g(a)$, then the succession (f_n) converges towards g.

But this leads us to another question: if the succession of derived functions (f'_n) converges toward the function g and if $f_n(a)$ has a limit, will there be a function g toward which the succession (f_n) converges and which has the derivative g?

The problem is difficult but worth attacking. A decisive observation is that with the succession (f'_n) converging toward g, the f'_n of fairly large indexes will be close to each other; but then, thanks to the theorem of the mean and the hypothesis about the succession $f_n(a)$, will the same be true for functions f_n of large indexes?

Does this imply the convergence of the succession (f_n)? This leads to posing the problem of Cauchy's criterion for the succession of real functions which is reduced to Cauchy's criterion for the succession of real numbers and which can be easily deduced from the axiom of nested intervals to which it is equivalent. In conclusion, the question posed is given an affirmative answer.

But then another question arises: Since step functions are among the simplest of derived functions, we wonder what functions may be approximated -- in the sense of the norm -- by step functions.

It is clear that a continuous function containing a given point may be approximated in a convenient interval such as ε by a constant function. Then can it be approximated in norm, that is, in all points a, by a function in steps? The positive response is given by the Borel-Lebesgue lemma for a closed interval [a,b]; this, provides the motivation and the proof can be immediately deduced from the existence of majorant minimum for every extended set of R. (Consider the set E of those $x \in [a,b]$ such that the segment [a,x] may be covered by a finite number of intervals of a given family. E is not null. It is increased by b.Let c be the majorant minimum of E. There exists an interval $c - \eta_1$, $c + \eta_2$ of the family. But $[a, c - \frac{\eta_1}{2}]$ is by hypothesis covered by a finite number of intervals of the given family, consequently also $[a, c + \frac{\eta_2}{2}]$. It is therefore impossible that $c < b$. We have $c = b$ and the reasoning infers $b \in E$.

It will be noted that the calculation of the primitive of the step function approaching a continuous function involves the so-called Cauchy-Riemann sums.

We have obtained the interval of continuous functions. Their linear character has been shown. The theorem of the mean may be expressed by

$$\left| \int_a^b f(t)dt \right| \leq ||f|| \; |b-a|$$

which means that the linear form (linear function)

$$f \longrightarrow \int_a^b f(t)dt$$

on a normed vector space C([a,b],R) of the continuous real functions of [a,b] is continuous.

New problems arise: to find all the linear and continuous forms over C([a,b],R). The answer is given by F. Riesz theorem. Beyond that, to discover the reasonable extensions of this linear form which leads us to the Lebesgue integral starting with the observation that $\int_a^b |f(t)|dt$ is a norm over C([a,b],R) for which the space is not complete.

Conclusion

Undoubtedly the last few presentations which I have described rapidly should be carried out in the first years at the university level. But the essential thing is that they have been carefully prepared for high school instruction. It seems clear to me that by emphasizing the structures involved and presenting each stage as the answer to a problem (but an answer which actually arises from others) we can achieve a coherent exposition which illumines the basic facts and stimulates the mind to its greatest activity.

The motivation for each step is instead of allowing mathematics to look like a collection of results unloosed, show it for what it is: a practical and conceptually rational answer to important problems.

ANALYSIS PROGRAMS IN THE UNIVERSITIES OF CENTRAL AMERICA

Eduardo Suger Cofino (Guatemala)

The specific theme assigned to me for this paper by the Organizing Committee of the Second Inter-American Conference on Mathematics Education refers to the programs of analysis followed in Central American Universities. Nonetheless, I believe it necessary to make some reference to our overall programs since on the one hand we must not loose the opportunity to profit from the help of continental colleagues, their ciriticism and suggestions for constructive modification; on the other hand, it is my belief that the inadequacy of our analysis programs is an immediate consequence of our general programs.

The programs confronting mathematics education in our region are very similar and therefore I shall not be ignoring the general situation by focusing in detail on the case of Guatemala.

General Considerations about Central American Universities

In chronological order, the first Central American university was founded in Guatemala in 1676; then came the University of Nicaragua 1812, El Salvador 1841, Honduras 1847, and Costa Rica 1940. The University of Costa Rica is really older than suggested; the old University of Saint Thomas was closed in 1888 and there was no university in Costa Rica until 1940 when the present University of Costa Rica was organized. Most of these universities, then, are 100 years old.

From the beginning, the greatest efforts have been almost always concentrated in social and juridicial science, medicine, pharmocology, and civil engineering. By contrast, the teaching and development of pure science and consequently technology was neglected. This has made economic development of the region difficult.

With regard to the development of mathematics, we can say that the concern for the intensification of the courses and the raising of the level of teaching in the university is recent. In this movement the recommendations of the Bogota Conference have had great influence. The common problem in the Central American universities with respect to mathematics instruction is the very poor preparation (worthless education limited to the mechanical manipulation of algebraic expressions and the resolution of triangles) which a student has as he emerges from the high school. Notwithstanding the reference made by the professors of university mathematics, the intermediate level has not reformed its study plans. I think it should be mentioned that the following professors have worked on the training of the intermediate level teachers and the formulation of elementary texts for mathematics: In Costa Rica, Professor Bernardo Alfaro, in Honduras, Professor Salvador Llopis, in El Salvador, Professor Carlos Umana, and in Nicaragua, Professor Roberto Zalaya Blanco.

Nothwithstanding these efforts, countries closely linked by historical-geographical and economical ties have not yet achieved unification of a minimal level of mathematics achievement in the high school, which would form a necessary basis for developing suitable plans for mathematics at a satisfactory university level.

One of the bases for effective advancing in every field of human knowledge is sincerity with one's self and loyalty to the scientific community to which one belongs, which, because it is scientific, recognizes no racial, political, or religious boundaries. It is for that reason that I am not embarrassed to say that we are still in the stage of early infancy in mathematics and we need fraternal assistance of the International Mathematics Community to guide us so that in a not too distant future you can count on us as effective members of the progress in mathematics and mathematics education of the domain of pure science.

Mathematics Programs in the General Study Programs

Central American University (aware of the inconvenience of a premature polarization of the university studies of a young man graduated from a weak high school, aware of urgent need for modern man to have reasonably solid knowledge in the various areas of human understanding which is not to be isolated in one artistical scientific community) created the area of general studies. It lasts between one and two years among the various countries.

Within this type of study, I shall detail the Guatemalian programs and by discussing the particular I shall be of some help to the general.

The student who enrolls in the general studies is 18 years old and generally has the impression that mathematics has a great deal of formulas which man uses to solve problems of daily life as finding the area of a piece of ground, the volume of ... the interest on capital and some other problems. The student has no intimation of the existence of mathematics for its own sake; that is, quite apart from fields of physics and engineering. It is also important to understand that we are dealing with a rather heterogeneous group of students, that is, they are future lawyers, doctors, scholars, etc.

Our problem then is to introduce the student to mathematics and show him that men, by doing mathematics, deal with universes which they can govern and for that very reason the mathematician is a maker of universes in which he defines the entities and postulates the properties and fundamental relationships. It must also be made clear that mathematics is useful (useful for "other things"), and therefore serves ends beyond itself whereby it is applied to physics, economics, physiology, etc.

The program that was operating in 1964-65 is here set forth so that the reader can gage the thoroughness of the course:

<u>Mathematics I</u> (9 hours weekly for one semester)

1. Introduction to symbolic logic. Operation of propositional truth tables, such as $(p \vee \sim q) \wedge (\sim p \vee q) \Longrightarrow p \vee q$
2. Elements of the Theory of Sets. Proof of formulas for equality such as $C \setminus (A \cup B) = (C \setminus A) \cup (C \setminus B)$
3. Functions. $F: A \to B$, injection, surjection, and bijection, inverse sum, product, composition.
4. Binary Relations. A relation R defined by $A \Rightarrow R \subset A \times A$; reflective, symmetric, transitive, antisymmetric, equivalence, equivalence classes, partitioning of a set, relations of order.

5. Algebraic Structures. Binary operation defined on A: $f: A \times A \longrightarrow A$; monads, semi groups, rings, integral domain, fields and theorems.
6. Integers. Construction and properties from the partitioning of $Z = J \times J/_{\sim}$ when J is the set of naturals and "~" is defined in $J \times J$ by: $(a,b) \sim (a'b') \iff a + b' = a' + b$, $a, a', b, b' \in J$. The structure $(Z,+,\cdot)$ as an integral domain.
7. The rational numbers. Construction and properties as a partitioning of $Q = Z \times {}^{Z}/_{\sim}$. The structure $(Q,+,\cdot)$ as a field, etc
8. The field of Cauchy classes
9. Metric spaces
10. Topological spaces
11. Limits of a function and continuity.

For students who may study engineering after general studies, a course called mathematics 3 was given with the following syllabus

<u>Mathematics III</u> (6 hours weekly, one semester)

The usual topology of real numbers

Generalized Intervals

Function of increments

Function of increment quotient

Derivatives

Polynomial, exponential, logarithmic and trigonometric functio

Fundamental theorems of differential calculus

The concept of an integral

Taylor's Theorem

Evaluation of Results obtained in the Mathematics Program of General Studies

We can say that about 8% of the 1000 students who went through this program showed excellent achievement. As a positive result of the program we may point to the fact that the student learned mathematics structures and was taught that starting with certain basic concepts it is possible to construct the entire mathematical edifice. As a negative result of the program we can point out that the students did not take the opportunity to become familiar with the intuitive aspects of mathematics and that since in courses 1 and 2 we had future doctors, biologists, veternarians, etc., for whom the intuitive aspect of mathematics is basic, we are dealing with a serious deficiency. Another serious defect of the program is due to the fact mentioned previously that the high school does not equip the student with tools such as analytic geometry so that the prospective student going through through the mathematics program enrolled,without even any knowledge of the equation of the ellipse in his specialty which made it impossible for him to take courses such as topography, fluid mechanics, etc.

Following a thorough analysis of the programs previously described and aided by the valuable conciliation of Jorge Arias (Rector of the University of San Carlos of Guatemala) and Dr. Burton Jones (regional specialist in mathematics in the CSUCA-NSF-AID Program) we selected the following programs:

Mathematics I

Topic I Sets. Operations on sets, Relations (Chapter I)

Topic II Relations of equivalence. Binary operations. Number systems. The rational numbers. Relations of order. The rational numbers as an ordered field (Chapter II)

Topic III The real numbers. Complex numbers (Chapter III)

Topic IV Absolute value. Graphs. Functions (Chapter IV)

Topic V Systems of linear equations (Chapter V)

Topic VI Exponential function. Logarithmic function (Chapter I

Topic VII Trigonometric functions (Chapter X)

The chapters refer to University Freshman Mathematics, Taylor-Wade, John Wiley and Sons.

Mathematics II

Topic I Mathematical Induction and the Binomial Theorem

Topic II Polynomials

Topic III Exponential and Logarithmic Functions

Topic IV Trigonometric Functions

Topic V Inverse Trigonometric Functions

Topic VI Trigonometric representation of complex numbers

Topic VII Solution of triangles

Mathematics III

Topic I Analytic Geometry of the plane

Topic II Functions. Sums, product, composition of.

Topic III Transformations

Topic IV Equations. Two variables. The conics

Topic V Limits and Derivatives

Topic VI Fundamental Theorems of Analysis

Topic VII Applications of differential calculus.

Recommended text: Intermediate Analysis, Haaser-Lasalle-Sullivan.

Mathematics IV

Topic I The definite Integral

Topic II Methods of Integration

Topic III Applications of Integral Calculus

Topic IV Taylor's Series

Topic V Functions of n variables

Recommended text: same as for Mathematics III.

The results obtained here is already satisfactory, in as much as 46 percent of 2000 students going through the program can be said to have received satisfactory results.

General Comments

a) Guatemala: As can be clearly seen from the foregoing exposition, we are still working on the formulation of general courses which include algebra and analysis. We do not yet have a specific algebra or analysis besides being troubled with a serious problem of not having a course in general geometry.

For 1967, the creation of a mathematics department is planned which would give us a chance to give a course in analysis perhaps at the level of the text by Rudin.

b) Honduras: The situation of Honduras is similar to that of Guatemala, since there is no course in analysis except for the elementary ideas offered in the program of general studies.

c) Nicaragua: Nicaragua (Leon) does have a Department of Mathematics which offers a graduate degree in mathematics and hence does offer formal courses in analysis. In the University of Managua, a course in analysis is not yet offered as a continuation for a course in the introduction to infinitesimal calculus.

d) Costa Rica: This is undoubtedly the Central American country that has a formal curriculum in analysis. It is given by the Department of Mathematics to future graduate students in mathematics.

D. On Teacher Education

THE RETRAINING OF TEACHERS IN PUERTO RICO

By Mariano Garcia, University of Puerto Rico

Mathematics education in Puerto Rico has undergone several changes in this last decade, especially during the last 5 years. We have undertaken numerious projects related to the revision of study plans for high school and beginning university instruction as well as retraining of mathematics and science teachers. As one might expect, the process has been gradual, but the achievements to date justify optimism for the future.

We have given particular efforts and attention to the retraining of teachers. Since 1957, supported by the NSF, more than 40 science and mathematics institutes have been held, with the participation of more than 1200 elementary and high school teachers. The Department of Public Instruction of Puerto Rico has given strong cooperation and support to this program of teacher improvement. The institutes have been basically of 3 kinds: Summer institutes, institutes for teachers in active service, and institutes during the school year.

The Summer institutes are usually conducted on several levels according to the background and needs of the participating teachers. Some teachers receive training in modern courses of biology, chemistry, physics and mathematics formulated by the various experimental curriculum study groups in the U.S. Others take special subjects such as statistics, mathematical analysis, and abstract algebra, disciplines which offer the participants a better understanding of mathematics and thus gives them the opportunity of teaching on a higher level. Participants may take 6 credits in mathematics and science. About 70 teachers from various cities of Puerto Rico and other American countries take part in each institute. We have had the privilege of having in the programs teachers from Bolivia, El Salvador, Nicaragua, Domician Republic, Panama, Honduras, Guatemala, Peru, Colombia, and the United States. Some institutes have been organized for the elementary school teachers, but the majority have been given at the high school level. The summer institutes generally last six to eight weeks and are held in June and July; that is, during the vacation period of the Puerto Rican school period.

The second type of institute is the one for teachers in active service. These are held during the school year, from August to May. Generally they are held on Saturdays in order not to interfere with the normal program of participating teachers. During three hours each week the teachers take modern science and mathematics courses designed to improve their skills and knowledge. Some of the courses tend to enrich the content of the mathematics and science courses that have been prepared by the experimental curricular studies group. Through these institutes, the participating teachers have a chance to take six credits during the school year.

The third type of institute, the school year kind, provides intensive training during the school year for small groups of carefully selected teachers who may become leaders in the modernization of study plans for mathematics and sciences in the entire island of Puerto Rico. They have also included participants from various South and Central American countries. Every participant has the opportunity of taking about 36 credits in mathematics and science. In Puerto Rico the school year institutes are generally given on an undergraduate level and include courses in mathematical statistics, modern algebra, elements of mathematics, modern geometry, eletronic computer programming, and general modern physics. Some of the better prepared teachers register for graduate courses leading to the MS degree where they major in mathematics. Every participating teacher in the school institute receives a maximum stipend of 3000 dollars over a period of 10 months plus 450 dollars for each dependent up to maximum of 4. In addition he is given 75 dollars for books and the cost of 2 round trips between his home and the university. He is also exempt from tuition cost.

The formal program of the institute is supplemented by special lectures by outstanding mathematicians from Puerto Rico, the U. S. and other countries as well as movies on mathematical and scientific subjects. Following are a few of the lecturers which we have had to date: Jean Dieudonne, of France; Alexander Dinghas, Wilhelm Maak and Max Deuring, of Germany; Kert Hirsch, of England; Lucas Bunt, Adrian Zaanen and Frederik van der Blij, of Holland; Hans Tornehave, from Denmark; Jose Tola Pasquel, of Peru; Howard Fehr, Oystein Ore, Cletus Oakley, Irving Adler and Truman Botts of the U.S.

In selecting teachers for each kind of institute consideration is given to academic average years of experience, previous education, and geographic distribution of the participants. The University of Puerto Rico is to continue to offer these and other programs for teachers in future.

Another program that has had great impact on teachers and students at the high school level in Puerto Rico is the program of lectures on mathematical and scientific subjects sponsored by NSF. These lectures are given by university professors and seek to broaden the teachers knowledge of the subjects they teach and to interest students in the field of mathematics and science. The following were among the subjects dealt with in the lectures: Introduction to modern algebra, groups, vectors, recreational mathematics, prefect numbers, geometric transformation, inequalities, mathematical system, matrices, sets, relations and functions, shortcuts in numerical computation, introduction to topology, mathematical logic, the real number system, mathematical induction.

Another program has been in operation since 1961 in the University of Puerto Rico, sponsored by NSF and also related to retraining of teachers. This is a special mathematics program for outstanding high school students. A university level course in mathematics is given with the goal of permitting participating students to undertake advanced mathematics study while finishing their last years of high school. The classes meet twice a week in a university lecture hall, generally 5 to 6:30 P.M. Here also, high school teachers participate in the program to observe at close range student reaction to the new concepts and so to familarize themselves with the new viewpoints on mathematical instruction at the university level.

In order that the teacher's participation not be totally passive they are given the duty of grading the students daily written homework. In addition some summer programs have been offered for outstanding students.

Three years ago, the Department of Instruction of Puerto Rico received a grant from the Ford Foundation for establishing a system of curriculum centers in various parts of the island. Three centers were organized one in each of the principle cities. These centers have already made the following contributions:

(a) The training of teachers by seminars, lectures, professional meeting, and special mathematics and science courses. *This aspect of the work is assisted by the collaboration from senior professors from the various universities of Puerto Rico who serve as consultants. These seminars are generally held twice a month and involve the teachers of the towns and the areas of the center.

(b) Preparation of curriculum materials for the enrichment of mathematics and science courses. Here we utilize the teachers working in the centers. They come from elementary and secondary schools and have usually received training at the various sciences and mathematical institutes. Once the materials are prepared they are reviewed by university level consultants and the directors of the mathematics programs of the Department of Instruction of Puerto Rico.

These projects have contributed substantially to the improvement of education and re-training of mathematics and science teachers in Puerto Rico, but the situation still requires attention and study.

In view of this, our plans are to continue offering the program described above and to enlarge as much as possible the activities related to the training of teachers and the continual revision of study programs. To this effect, a commission of mathematics has recently been created with the support of the College Entrance Examination Board Office of Puerto Rico to carry out research and studies about mathematics instruction in Puerto Rico at both the high school and university levels. These studies will be carried out during the next two years with a goal to institute in Puerto Rico an experimental academic program in mathematics. It is hoped that the work of the commission will result in significant improvement in mathematics instruction in Puerto Rico.

The above presents, briefly, the various programs related to the retraining of teachers, as they have been developed in Puerto Rico during the last decade. These efforts have given improvement to high school instruction in our system. However, we are not completely satisfied by these efforts and we shall continue exploring other possible avenues. A large group of teachers has not yet benefited from these programs, since they have not participated in the institutes and other programs. And the incorporation of new topics into the study plans of high schools complicates the situation even more. The teachers require training in the instruction of these areas and need the opportunity to meet scientists and mathematicians of reputation who can encourage and help them in the presentation of new material to their students. In Puerto Rico we remain continually alert to new ideas, endeavouring to improve the knowledge of our teachers and keep them continually up to date as the study plans continue to change. Our goal is a faculty of the highest academic and professional competence.

TEACHER TRAINING IN BRAZIL

Martha Maria de Souza Dantas

I am honored to give a talk on the training of high school teachers in Brazil. This is doubly difficult, since there is the matter of representing a country, in which territorial extention, regional differences in climate, race, and economic and human resources, make a uniform plan difficult. To handle this project properly would require investigating the actual situation in every state of the country, a procedure which in Brazil would require personal interviews. Thanks to the assistance of the Directory of the Federal University of Bahia and the Mathematics and Physics Institute, two teachers have visited some of the state capitols and have gathered some information. Hence, while it is impossible to generalize, this report will limit its statements to dominate aspects.

For the sake of clarity, we shall treat the subject of trainin of high school teachers in brazil under three headings: the concept the need, and the procedure.

The Brazilian Concept of the Training of High School Teachers

a) By teacher training in Brazil we mean the training given in the schools of philosophy. It consists of a series of practice experiences included in the subject of special mathematics educatio

b) We refer to the preparation of the teacher for handling new methods of mathematics instruction which are being disseminated through courses which are also called teacher improvement courses.

The Need for Teacher Training for the High School Level

Since it is not possible to understand a work plan without some knowledge of peculiarities of the atmosphere to which it applies, I shall preface the central considerations with a few observations.

As was indicated in the report which we presented to this Second Inter-American Conference on Mathematics Education, our first national meeting of high school mathematics teachers took place in 1955, focusing primarily on the distribution of subject content. When the compartmentalization goal had been reconstructed we turned our attention to reformulating instructional methods in an attempt to make them less formal. But the program was still not up to date. The dissemination of the findings of the "Commission Internationale pour L'Etude at Amelioration de L'Enseignement des Mathematiques" showed the way for high school level instruction to be in line with contemporary mathematics. "It was necessary to avoid the useless sacrifice of our young people who, upon entering the university, had to reclassify the body of their knowledge in the light of different ideas and a different language which also introduced new thought." This sentence shocked many teachers who did not believe in the applicability of modern mathematics to the high school level, but it drew admiration from the majority of Brazilian teachers, some of whom came from university backgrounds, others from schools of philosophy, science, or liberal arts which taught only a few mathematical topics of the 19th century. We sensed the incompetency of traditional instruction.

and felt completely unprepared for modern teaching. We felt the we of our responsibility. In 1957 in the second National Meeting the theme was: Traditional or Modern mathematics at the high school level? But how could we answer that question when some teachers also asked "What is modern mathematics?"

At the third National Meeting in 1959, severe criticism was voiced of the mathematics education given in the schools of philosophy - even the very best - and among the conclusions of the meeting we included a request that the Ministry of Culture and Education study the new structuring of mathematics courses in the schools of philosophy. It was also requested that these schools include in their curricular education a study of modern mathematics for high school teachers.

By the third National Meeting we had taken a position with respect to the situation of mathematics instruction in Brazil that revealed an appraisal of teaching personnel, namely we were completely behind the times.

Two years later, in 1961, in Bogota, Colombia, Professor Omar Catunda of Brazil, in his talks on "Training of Mathematics Teachers in Brazil" explained that although this was a function of the schools of philosophy, sciences and liberal arts, only a small proportion of teachers receive this education. He recommended that due to the lack of teachers for the many public schools (and that number is increasing) the authorities officially authorize certification of teachers without demanding specialized preparation.

In this respect, no change has taken place between 1961 and the present. The law of directions and bases for national education passed in December 1961 continues in force in the country. Consequently the preparation of high school teachers continues to be the function of the faculties of philosophy, science and liberal arts and the certification of teachers continues without specialized preparation.

It should be added that many states of the union do not have mathematics courses in the School of Philosophy, where such schools exist. The law of directions and bases vetoed the article that required examinations for the position of teacher in public schools. In general, there are no provocation procedures for the country's high school teachers to stimulate them to study.

While the education of high school teachers in Brazil remains so problematical, national and international insistance continues for better instruction to prepare today's young people for the requirement for tomorrow.

Hence, the unpostponable task of training teachers in Brazil is too complex since it cannot be limited to imbuing faculty with the spirit of today's mathematics. Then the majority of cases it will be necessary to furnish special education, in some cases to correct inadequacies, in others, to fill an absolute vacuum.

Current Procedures in Brazil for the Training of High School Teachers

As is again indicated in the report we are presenting to the second Inter-American Conference on Mathematics Education, our first teacher training courses were given in Sao Paulo January 19 by professors of the university and the Sao Paulo State Office of Education. These courses lasted 20 days each, the first one, call first stage, included the following topics:

a) Mathematical logic (8 hours)

b) Set theory (8 hours)

c) Practice (18 hours)

d) discussions (3 hours)

The second course, called second stage and of a more advanced leve included:

a) modern algebra (9 hours)

b) solid geometry (8 hours)

c) practical application (9 hours)

d) lectures (2 hours)

e) discussions (3 hours)

These courses were given in conjunction with the Ministry of Education and Culture of the Sao Paulo Office of Education.

At the end of each course the participants were given aptitude tests. Courses such as these were repeated in the following years, almost always during vacations. From January to February of 1965, the third stage was given, including modern topology, linear programming and seminars.

In Rio Grande del Sur, the center of educational research orientation (CPO) has a plan of action for modernizing mathematical instruction in its various fields within the next years. This year, in the high school teaching sector, a course being given from April to December with 5 hours per week. We not yet have information on the program. In general they have given the vacation courses and programs for modernizing the knowledge of high school teachers.

There is also some concern for preparing teams for implementing plan; they will be made up of university faculty with graduate degrees. For this course which is already in operation, the following subjects are scheduled: matrices and linear algebra, general topology, theory of numbers, algebraic structures, vector spaces, mathematical logic, and Boolean algebra.

In Porto Alegra, capitol of Rio Grande del Sur, the problem teaching personnel for high school mathematics is less acute than in other capitols of the country, because the state law provides that only teachers with graduate degrees in mathematics may teach in the state schools, and to date the number of such qualified teachers has been adequate for the needs. The problem of specialized personnel is serious in the interior, as well as in general throughout the country.

In Rio de Janeiro, the old capitol, because of the dense population and the insufficient number of graduate degrees in mathematics emerging from the National School of Philosophy, as well as private schools, we can assume that there is great difficulty in modernizing the teaching of the subjects desribed above. We may point out that in the past five years the largest group of graduate degrees in mathematics occurred at the National School of Philosophy which, in 1964, was made up of 20 graduates.

In order to promote the modernization of mathematics teachers in Rio de Janeiro (Guanabara state), the center for mathematics teacher training at Rio de Janeiro was founded by an agreement between the Pontificial Catholic University of Rio de Janeiro and the Ministry of Education and Culture. It began operations in April of this year. This center is offering, in the second semester of 1966, a course in basic ideas of set theory, providing 10 one hour classes in theory. Every theory class is followed by an hour of discussion and directed study. Two courses are planned: one on the introduction to linear algebra, the other on elements of mathematical logic, each one also made up of 10 classes.

In Bahia, the teacher training courses in mathematics scheduled for 1958 did not begin until February 1964 and then only thanks for support from the Office of the Superintendent of Development of the Northeast (SUDENE). These courses correspond to programming set up by the high school teachers under the guidance of the Institute of Mathematics and Physics of the Federal University of Bahia. This programming assumes the needs for the at le four basic subjects or studies organized as follows: first stage, elements of symbolic logic, introduction to set theory, principle algebraic structures (notions), practical applications;

second stage: modern algebra, line and plane geometries; third stage: solid geometry and the study of matrices; fourth stage: elements of topology, differential and integral calculus. Each one of these stages has a minimum duration of one month. Provision is made for at least 64 theory-practice classes as well as the same number of directed study classes for each of the stages. To date in view of the antiquated conditions of instructor preparation in Bahia, the first stage has been given five times. The teachers are tested, and registration in the following stage is dependent upon prior passing of the previous one. The giving of the second stage is scheduled for July 1967.

Last year, the Ministry of Education and Culture, feeling the need to participate more directly in the training of basic science teachers in the country, joined with the universities and offices of education to create five centers called Centers for Science Instruction. They are located in the following states: Rio Grande do Sul, Sao Paulo, Guanabara, Minas Gerais Pernambuco, and Bahia. With exception of the Guanabara Center, there are now mathematics sections in operation in each of these centers. In Bahia, for example, the creation of the instructional center for sciences permitted a continuation of the teacher training courses for the mathematics teachers which had begun in 1964, but which had been interrupted in 1965. The Northeast Centers, that is those of Bahia and Pernambuco enjoy the assistance of the SUDENE. Because of this, teachers enrolling in such courses receive monetory assistance in the form of scholarships In Bernambuco and

and Minas Gerais the training courses for mathematics teachers a also given by the instructional centers for sciences. In these s although the programs are not the same, the training is also of one month duration. We have no information about any long term programming for this project. As to the subjects, the following topics have been taken up: elements of set theory, mathematical logic, probability, modern algebra and linear algebra. The Pernambuco courses also provide directed study and aptitude test

In the state of Ceara, teacher improvement is one of the go of the Mathematics Institute of the Federal University of Ceara. In this state, two courses of modernization of high school teaching were given, one in 1964 in conjunction with SUDE, another in July 1965, sponsored by the Fulbright Commission. In the former, the first volume of Studies in Mathematics, a translation of the text of the SMSG, published by IBECC, was stu In the second, volumes two and three were used. In the state of Paraiba, also, courses are now given to train mathematics teache at the high school level.

As may be gathered from this description, while there is no general uniform plan for the preparation of high school teach there is a tacit understanding about how this work should be undertaken and there is a more or less common basic education. Fo example, all the early courses deal with elements of set theory and some mathematical logic. The logic course seeks primarily to equip the Brazilian mathematics teacher, who generally does not know how to deny a proposition, so that he can fruitfully begin the necessary study for bringing himself up to date. Hence, the course emphasizes the study of deductive procedures. Also is a great deal of enthusiasm for the practical applications as modern practice. This satisfies the teacher's curiosity showing him how modern mathematics works in the high school se. This objective is achieved by the modern treatment of the and their program. Modern algebra, which is still not a ulsory subject in some schools because it is considered too ract, must be approached with care. The syllabus for this se must be chosen carefully, selecting that which constitutes best contribution for the high school teacher's education. seems to us premature to introduce linear algebra in the first stages. We speak as Northeastern teachers, from a part of the try where the problem of teaching and other specialized personnel considerably graver than in the Southern part of the country, example. Our experience in the courses given in El Salvado - ia, leads us to the conclusion that even in a logic course with ess than 18 hours of theory-practice classes, it is impossible to elect the indispensible knowledge of calculus, propositions, study quantifiers and formal equivalence. Similarly with less than 20 theory-practice hours one cannot manage to show that a Boolean algebra structure is defined by the operations of intersection and union in the total parts of a given set. It must be stated that the Salvado courses are always given during school time and that every theory-practice class is matched by at least one hour of directed study guided by a competent professor. Also the teachers take aptitutde tests every week. Our experience has shown that it is impossible to make responsible judgments as to how the teacher benefits without giving aptitude tests. We can only guarantee profit from the courses through pedagogical handling and statistical evaluation of aptitude tests. It is of no help to give more complex courses, if earlier difficulties have not been surmounted. Most of our teachers need, above all, to overcome the deficiencies of their education; that is to learn to reason well, to abstract and generalize, and hence to be able to receive new information. For this work it is necessary to always bear in mind the utilitarian mentality that dominates instruction and produces the question, "What is modern mathematics good for?"

Hence it is necessary to take great care in preparing everything that will be presented to the teacher, since the instru tion which is too abstract may at any time discourage him for good

In general terms, these are the conditions surrounding the training of mathematics teachers in the high school level in Brazi

We should, however, express our thanks to the assistance of t teachers from some schools of philosophies and mathematics institu in our country. Access to mathematical colloquiums granted to high school teachers permits greater understanding between the teachers of the two levels and consequently the reformulation of goals at each level for the education of today's youth and tomorrow's mathematics. Meanwhile this work which is practically one of salvaging is costly and painful to the country and hardly an ideal lution. The proper solution would undoubtedly be reconstructing from the bottom up a specialized education for mathematics teachers conveniently organized and up to date.

A RIGOROUS TEACHER TRAINING PROGRAM IN WESTERN GERMANY

Hans-Georg Steiner
University of Münster, Germany

1. School System:

In Western Germany as a federal republic the responsibility for cultural and educational affairs belongs to the Government of the "lands."* We have differences in many details of our school system, varying from "land" to "land." There is a common background, however, of experience and tradition which has a great unifying influence.

The following scheme demonstrates the general struct of school education in Germany.

About 50% of all children attend one type of school only, the "people's school," which is now a period of 9 years After that the pupils usually take an apprenticeship and then have to attend a vocational school ("Berufsschule") for at least two years.

There are two types of secondary schools. In some "lands" these start at grade 5, in other "lands" at grade 7. The Mittelschule corresponds to the British secondary modern school. It's orientation is not as theoretical as that of the "Gymnasium", which leads to University. Having successfully

*"lands" is comparable to "states" in the U.S.A.

<table>
<tr><th>Grade</th><th>Age</th><th colspan="5">Type of School</th></tr>
<tr><td>1
2
3
4</td><td>6-7
7-8
8-9
9-10</td><td colspan="5">"GRUNDSCHULE" (Elementary School)
1st Part of "Volksschule"</td></tr>
<tr><td>5
6
7
8</td><td>10-11
11-12
12-13
13-14</td><td>2nd Part of "Volksschule" 50%</td><td rowspan="3">"Mittel-Schule" (Middle-School) 20%</td><td colspan="3">"GYMNASIUM" (Grammar School) 30%</td></tr>
<tr><td>9</td><td>14-15</td><td></td><td colspan="3" rowspan="2">Division into 3 or more Branches</td></tr>
<tr><td>10</td><td>15-16</td><td></td></tr>
<tr><td>11
12
13</td><td>16-17
17-18
18-19</td><td colspan="2"></td><td>Classical</td><td>Modern lan-guages</td><td>Mathematic Science</td></tr>
</table>

finished the Middle School at the end of the 10th grade, pupils get a certificate which entitles them for instance to proceed to a technical school or a higher commercial school. At the end of Gymnasium education there is an examination, th so-called Abitur, which in general is necessary matriculation at a University of any institute of the same rank, as the Technische Hochschule or Pädagogische Hochschule.

From grade 9 on - or even earlier - there is a ramification within the "Gymnasium" into three or more branches, but the instruction in these branches is based on the same principles and comprises the same group of subjects. The difference shows itself in the emphasis laid on either classic or modern languages, or mathematics and natural sciences, or - as a rather new branch - social sciences.

2. Mathematics in Schools:

Mathematics is taught in nearly every form of German schools. Exceptions exist only in some "lands" where a Gymnasium pupil is allowed to finish his mathematics educatio at the end of the 12th year in order to concentrate on other fields during the final school year.

In the "Volksschule (8 or 9 years) mathematical instruction is confined to Arithmetic and Geometry insofar as it serves practical purposes in daily life.

Mathematical instruction in the "Mittelschule" (5 years) teaches a pupil elementary mathematical notions and methods and their application to practical life:

arithmetic, percentage, functions, linear and quadratic equations, graphing, logarithm and slide-rule, intuitive geometry, solid bodies, trigonometry.

Mathematical instruction in the "Gymnasium" is aimed at developing intuitive and logical thinking by giving the pupils an experience and knowledge of certain elementary parts of mathematics and applications and by leading them to some basic methodological insights. So it includes an introduction into calculus and its application to physics, to analytic geometry by means of vectors and transformations, and to algebra in the complex number field.

There are trends of <u>modernization</u> in all types of schools. The strongest trends are in the reform of mathematics education at the "Gymnasium": First three years of "Gymnasium": early introduction of negative numbers and variables (as place holders), sets, grammar of set language, simple combinational analysis, use of vectors, vector addition and mappings (reflexions, rotations, translations) in propedeutic geometry; preparation of the concept of group; more emphasis on relations, especially the ordering relation for numbers, inequalities. Next three years: extensive treatment of functions, their representations and use; improved treatment of equations and inequalities from a logical and semantical point of view; investigation of the group of plane congruence mappings and its subgroups, especially symmetry groups of figures; multiplication of vectors by rationals, dilations and similarity;

examples of groups, rings, fields, especially finite models. Last three years: introduction to modern elementary algebra as to the theory of algebraical structures, axiomatization of the real number system, vector spaces and scalar-product; foundation of plane and space geometry by means of linear algebra; introduction to probability theory and statistics; methodological aspects of mathematics.

3. Training of Mathematics Teachers:

Teachers of all types of school have to attend a "Gymnasium" and finished their school education by passing the "Abitur." Teachers at a "Volksschule" (elementary school teachers) get their training at a "Pädagogische Hochschule." They are expected to teach every subject and thus they usually get a specialized training in special subjects. Their mathematic background consists in what they have learned at the "Gymnasium. But they have to take part during their 3-year study in at least two one-semester seminars on teaching arithmetic or geometry. Besides this every student has to select one field after his choice in order to deepen his performance but not to specialize as a teacher in this field. This can be the field of mathematics. So the professors of mathematics education at the "Pädagogische Hochschule" offer lectures and seminars on mathematics in which they usually take up subjects of the "Gymnasium" curriculum or topics which are related to them and enrich these by additional material or new points of view. Those subjects are: sets and set-theoretical foundation of the cardinals, the number system, elementary group theory, foundations of geometry, non-euclidean geometry, probability theory, history of mathematics.

My point of view is that we need a specialized elementary school teacher in the field of mathematics. Didactical and mathematical analysis at the very elementary levels have detected a rich structure of learning and teaching even at a pre-mathematical level. To make use of these insights and in order to develop good mathematical imagination and thinking in a child as effective, and as early, as possible, we need teachers with a special preparation in mathematics and also in modern mathematics teaching.

The training of the "middle school" teachers who are specialized in two fields differs in different "lands." In some "lands" mathematics teachers at a "middle school" have to study 6 semesters at a university. After that he has to pass an examination both written and oral in mathematics and likewise in another field which very often is physics studied during the same time. The topics of this examination are usually taken out of the standard university lectures on calculus, analytic geometry and linear algebra, elementary number theory, elementary modern algebra and foundations of geometry. If the university offers special lectures or - what we call - didactics of mathematics" the examination can also include questions on elementary mathematics from an advanced point of view. This will be explained in greater detail in my comments on the training of the "Gymnasium" teacher.

Having finished his study at the university which at present lasts 4 years instead of the required three years, the "middle school" teacher is a participant of a preparatory seminar for an additional one and a half years. Here he gets guided experiences in teaching middle school classes and instructions on theory and practice of education. The details of this can also be explained in connection with my comments on the Gymnasium teacher, because the Gymnasium teacher training has been taken as a model for this new training of middle school teachers.

In some "lands" there are possibilities for an elementary school teacher to be promoted to the position of a middle school teacher by taking courses in mathematics, which are held by teachers - usually outstanding Gymnasium teachers - in different cities. But the examination board for these candidates is the same as for those who have studied at the university and it has been found that only a very few elementary school teachers are able to graduate by merely taking such courses.

In other "lands" the only way of becoming a "middle school" teacher is following the line of an elementary school teacher and graduating at the "Pädagogische Hochschule." Thus there is found a great difference in the mathematical background of "middle school" teachers in Germany, depending on their training.

A prospective Gymnasium teacher has to choose at least two fields which are taught in the "Gymnasium" and he has to study these at the university. At present the time of studying for a mathematics teacher really is much longer than four years. At the university of Münster it is about 12 semesters or six years. Besides the lectures, exercises and seminars in these two fields, the student also has to attend lectures and seminars in philosophy and pedagogy. At the end of his study there is a rigorous examination, called the "First State Examination (Referendarexamen). The student can choose one of his fields as his major field.

If this field is mathematics he has to write a thesis of about 80 to 100 pages on a special mathematical subject by making use of all material available, the newest articles in the mathematical periodicals included. Besides this he has to pass one or two written examinations in each of his fields, done in the rooms of the office of examination in a restricted time of 4 or 5 hours each.

In addition to this there is a full hour oral examination in mathematics and another full hour examination in the second field.

A written and oral examination in philosophy and pedagogy, the so-called "Philosophicum," is a part of the "First State Examination." But it can be passed separately after the 6th semester.

After completing University study the prospective gymnasi teacher has to go through a two-year period of teacher prepara in the first year being connected with a school, in the second year being a member of a special "seminar," in which practice and theory of school life and teaching are combined. At the end of these two years there is another examination, the so-called "Second State Examination," (Assessorexamen). The "Referendar" as the candidate is called during these two years - has to wri a report on a certain period of his own teaching in a class, to analyze the mathematical and didactical structure of the topics taught, to describe the reactions of the pupils and his work with the pupils, to characterize individual features of some pupils, and to evaluate his teaching. Furthermore, he has to present two demonstration lessons in front of an examination board, one in a class known to him, one in a class not familiar to him. Finally there is an oral examination in pedagogy and methodology of teaching.

4. Mathematical Background of Mathematics Teachers:

The University study of a Gymnasium teacher in Germany last too long at present. This duration is a result of different influences, one of which can be recognized in the extension and complication of our scientific knowledge. To solve the problem of an adequate limitation, both in content and time, several commissions have been instituted. In particular I mention the commissions of the Union of German Mathematicians (Deutsche Mathematiker Vereinigung), the commission of the

[So]ciety for Applied Mathematics and Mechanics (GAMM), and the [co]mmissions which have been set up in the different "lands."

The proposals which have been made by these commissions [ag]ree in the following points:

improved organization of courses, especially for beginners, in order to enable the student to finish his study no later than at the end of his 10th semester, his thesis and final examinations included;

more emphasis on links between university mathematics and mathematics taught on the secondary school level; greater efficiency of the "Philosophicum" by replacing general philosophy by mathematical logic and foundations of mathematics and replacing general pedagogy by a specialized "Pedagogy of the Gymnasium" and didactic of mathematics.

The proposal of the commission of the Union of German Mathematicians says that the "Basic Study" in mathematics should be the same for prospective mathematics teachers and for those preparing for a mathematical profession in industry or economy. It suggests the following schedule:

Semester	Course	Weekly Hours
1.	Calculus I	4 + 2
	Analytic Geometry and Linear Algebra I	4 + 2
2.	Calculus II	4 + 2
	Analytic Geometry and Linear Algebra II	4 + 2
3.	Vector Analysis and Differential equations	4 + 2
	Topology or algebra	4 + 1
4.	Complex Analysis I	4 + 1
	Numerical Analysis and "Practicum"	4 + 2

In the column "weekly hours" of the scheme the first numb means the number of 45 minutes lectures in a week and the seco number means the number of 45 minutes weekly periods of workin in small groups, what we call "Übungen" (Practice). These grou are guided by assistants and tutors. The students work on particular parts of the course, on additional topics and problems, and discuss solutions of problems they have been working on at home during the foregoing week or fortnight.

The "Main Study" shall give the student more freedom for choosing fields after his interests. Those fields which are listed as relevant for a teacher are:

a) Analysis and Topology

b) Geometry

c) Algebra and Theory of numbers

d) Functional Analysis and Numerical Analysis

e) Probability Theory and Statistics

f) Foundations and Logic.

Every prospective Gymnasium geacher in mathematics is expected to have attended at least one course or more in each of the first four fields mentioned. If he has selected mathematics as his major field, he is expected to have taken four courses more in any of the 6 domains mentioned. He shall have attended two seminars and given at least one seminar paper. He also shall have attended lectures on didactic of mathematics and a didactic seminar.

In order to explain what is meant by the courses which belong to the "Basic Study" I will give a short list of topics which belong to the courses Analytic Geometry and Linear Algebra I, II: groups, fields, vector spaces, linear mappings, finite dimensional vector spaces, matrices, affine spaces, systems of linear equations, scalar product, euclidean vector spaces, eigen-value, eigen-vector, characteristic polynomial, normal-form of endomophisms and matrices, isomorphies, applications to classification of quadratics, fundamentals of projective geometry, the "Erlanger Programm," multilinear algebra, determinants.

5. Epistemology and Didactic of Mathematics:

In Germany many universities have institutes for foundations of mathematics and mathematical logic. At the University of Münster there is the oldest institute of this kind in Germany. Here for many years it is the convention that the students can take courses in methodology and logic in order to prepare for that part of the so-called "Philosophicum" which usually belongs to the field of general philosophy. For these candidates the institute offers a cycle of the following courses:

a) Predicate Calculus

b) Enumerability, Computability, Decidability

c) Naive and Axiomatic Set Theory.

The need for didactic of mathematics at the university had been stressed by Felix Klein. in 1908 he complained about what he called the "double discontinuity" in thecareer of a mathematics teacher, who has to jump twice over the ditch between university mathematics and mathematics at school.

Klein himself gave a famous example on how to work against this "double discontinuity" by offering lectures on "Elementary Mathematics from an advanced Point of View" and by his important contrubitions to a reform movement in the teaching of mathematic in the secondary school. As far as teacher training is concerned Klein did not find too many followers. In general, the situation he complained about became even worse. The courses for the beginners became more abstract and the axiomatic way of teaching at the university made it more and more difficult for a young teacher to find a way of using the things he has learned in his own secondary school teaching.

On the other hand a partial solution for this problem has been found in Münster and this is now being taken as a model by other universities. In 1952, following the ideas of Klein, Professor Behnke from the University of Münster set up a first "Seminar on Didactic of Mathematics" in Germany. This was an institutional continuation of some activities in the field of mathematical education in which Behnke had been collaborating with Otto Toeplitz before the war. In this Seminar, university lecturers, students of upper semesters, and teachers from the Gymnasiums of the city and the neighborhood work together.

The problems discussed during the first years mainly belonged to the theory and practice of Klein's reform movement. These problems were related to teaching calculus and applications in the upper forms of the Gymnasium and to teaching geometry to dren of age 10-12 at an intuitive and propedeutic level. e about 1956, the aspects of teaching sets, relations and ctural concepts as group, vector space, ordering, etc. and penetration of secondary school mathematics by these epts and new ideas became dominating. Moreover the basic tude of the seminar is reflected in the following point: entary mathematics has to be an adequate representation of ent time mathematics and of all the essential mathematical vities on an elementary level. Besides stating or con- turing theorems and giving proofs there are other activities experimenting, induction, mathematizing a situation, alizing, explicating intuitive ideas, axiomatizing, local global organizing, formalizing, codifying, etc. Everything ch could be of importance for building up this elementary hematics, reaching from the different standpoints in the ndations of mathematics on one hand to the newest applications any field on the other hand, should be taken into consideration.

By "Didactic of Mathematics" we mean the systematical nvestigation of the possibilities of presenting mathematics t different levels, of finding different structurations for mathematical content or proof, of initiative situations from where pupils can be lead to a problem or to a concept and so on. We became aware that didactic of mathematics has to be considered as a crossing point of different fields.

The seminar meetings in Münster always start with a paper given by participants or invited speakers: university lecturers, teachers, or students. The university lecturers usually contribute new theoretical considerations, the teachers, reports on their teaching experiences, and the students, reports on new publications or a report of the following type. A student is brought into contact with specialists in the social sciences and he studies applications of ordering theory within these sciences in order to select those problems which are both sufficiently elementary and interesting that they could be treated in a secondary school class. After every report there is a full hour of discussion.

Besides the seminar, a lecturership in didactic of mathematics has been in operation for several years. The lecturer who has teaching experience in the Gymnasium is specialized in the field of didactic of Mathematics. He offers lectures on the teaching of analysis, algebra and geometry and also on special topics such as the teaching of logic in the secondary school, the role of history of mathematics in teaching, modern applications of mathematics on the secondary school level, etc.

In analyzing the didactical structure, the didactic courses also touch levels of understanding mathematics which belong to the history of mathematics. In this way they try to give the student at least a brief insight into the outstanding historical situations which in fact should always be done on the basis of newest critical historical research.

It is evident that the didactic courses can be given on ferent levels. Most of the students appreciate the ventory and motivational point of view and prefer a course ch rearranges their mathematical knowledge by going through in a new way. But a progressive training of teachers has to ke care of the fact that the teachers themselves have to be oductive and have to be active in a continuous and not a ce for ever reformation period of mathematics teaching.

NEW TEACHER TRAINING PROGRAMS IN ARGENTINA

H. Renato Völker

Prior Considerations

The Argentinian school system still has a relatively long elementary school phase of seven years, followed by a secondary school of five years which is divided into the first or basic cycle of three years and a second cycle of two years, which may be of liberal arts or teacher preparation. Business schools are also five year institutions in which the first three years are equivalent to the aforementioned basic cycle at least as far as mathematics programs go. There are twelve school years, then, involved before reaching higher level or university instruction and in this regard the Argentinian school system differs very little if at all from other countries. Differences, however, are greater in the duration of elementary level studies which must be completed for entry into the secondary institutions. The results are that there is extra time in the seven elementary years and a shortage of time for completing the programs at the secondary level. And secondary programs in Argentina as in other countries, are generally rather full. The problem could be solved by transforming at least the seventh elementary grade into a first year of secondary level which would allow time for covering the corresponding material.

Unfortunately there were and still are considerations of education politics that make this transformation difficult and we must assume that the system of seven elementary grades and five secondary grades will have to be dealt with for the time being.

Given this fact, and the objectives of reform in mathematics instruction, a solution might be found by transfering a part of the secondary course to the sixth and seventh elementary grades; but then we face the problem of the inadequate preparation of the elementary teachers who have only a secondary level education in Argentina. (Since the normal schools from which they graduate after three years of a basic cycle and only two years of teacher preparation.) There is a noticable strong tendency to lengthen these studies, with the principle objective of improving the scientific preparation of elementary school teachers who to date are less prepared in this regard than the liberal arts graduate. Attempts might be made to bring the mathematics knowledge of elementary school teachers in service up to date but the great number of such teachers would make such a task very difficult and require a long period of time for particular results to be achieved. A realistic analysis of the situation implies, then, accepting for the moment the seven elementary and five secondary years and trying to extend the latter to six in the future. This system Argentina must accomplish if any reforms in mathematics programs occur at all levels, and it is along these lines that we are working.

The Common or Regular High School Programs

These programs presuppose that the student has learned in elementary school the four basic operations with natural numbers, that he can work with decimals and relatively easy fractions, operate with simple and compound three-step procedures, calculate percentage, surfaces and volumes of simple geometric figures, and know some of the properties of such figures. Entrance examinations at the first level of the high school show that candidates have acquired these skills in elementary school but on the basis of memorized rules in a mechanical setting. For this reason, there is currently a reaction against rote and attempts are being made to have mathematics taught on the elementary level from a rational viewpoint, teaching the student to use his intelligence to solve problems which are not amenable to the simple application of a formula or rule.

The regular or common high school programs presently in use provide some geometry, some arithmetic, and some algebra of a traditional nature in each of the first four years. In the fifth course trigonometry and cosmography are taught. Five hours a week are given to mathematics in each of the first two years and four hours for each of the remaining years. Geometry - plane in the first three courses and solid in the fourth - is taught in the Euclidean form. Trigonometry, includes the solution of triangles; cosmography deals basically with a positional astronomy and a little astrophysics.

The contents and sequence of these programs dates back to a reform of 1926, which in its time was warmly praised by Frederico Enriques and which constituted notable progress within the chaotic situation of that period. Nevertheless, it met with stiff opposition at the time and was able to be achieved only by the strength and technical competence of the author of that reform, Professor Florence D. Jaime and the Ministry which supported him fully. It is true that since that distant time there have been repeated changes but, with the exception of one short lived one, these changes did not affect subject content per se but rather the intensity and the extension of certain topics and the extent of thoroughness with which they are treated.

It was not until 1965 on the basis of good results obtained by the new experimental programs that it was decided to include in the common programs certain topics of current mathematics. I refer to elements of set theory, relations, functions, binary operations, inequations, vectors, statistics and probability.

Experimental High School Program

One of the results of Professor Marshall Stone's 1962 visit to Buenos Aires was agreement to publish mathematics programs for the high school cycle which would meet the demands of scientific advances and the recommendations of international conferences, especially the Bogota Conference. These programs were to be as advanced in reform as was suitable in our country. The task was placed in the hands of a group of professors of the Mathematics

Department of the Division of Exact and Natural Sciences of the University of Buenos Aires. The programs were devised for implementation in six courses which is the customary number in the high schools connected with the university, but they do not loose continuity if the last course is eliminated. One of the essential differences between the experimental and the common programs is the curtailment, in the former, of geometry and trigonometry in favor of algebra. Almost all traditional geometry is concentrated in the first course, but with an intuitive focus The starting point is the intuitive concepts of point, line, plane, length, surface, etc., as an abstraction of a visible ima concepts brought by the student from the elementary school or in this first course itself. These concepts are then amplified by observations and demonstrations designed to cause the student to exercise a critical mentality and rational capacity. Apparent the instruction in this geometry is not based on the axiomatic methods but rather leaves such methods to the algebra courses in the later years, since algebra is a better vehicle than geometry for introducing axiomatic reasoning and simple structure

The second course in the new program constitutes a decisive advance and establishes the bases for study in later courses. The program essentially has two stages:

(1) Set theory, relations, binary functions, and operations;

(2) Theory of numbers up to and including rationals.

Experience has shown that there is no difficulty in implementing the first stage of the program; but the shortage of time and the lack of mental maturity of the students makes it difficult to accomplish the work of the second stage through thorough axiomatic study. It was necessary to introduce a concept of natural numbers by means of the cardinality of sets.

For the third course, the program provides the study of real numbers and questions of elementary trigonometry; it includes topics of plane analytical geometry, vectors of two dimensions, linear equations, systems of equations and inequations, and graphic representations.

The fourth course takes up complex numbers, second degree equations, single variable polynomials, elementary functions, analytic geometry, sequences, elements of probability, and statistics and as a supplement some questions of business arithmetic.

The fifth and final course re-examines the intuitive geometry that was studied in the first course, but now with critical discussion of the implicit premises. It also includes the study of linear analytic geometry in space, using vectors and finally it deals with some questions of divisibility.

Beginning and Development of the Experiment

Once the experimental program had been formulated and debated, the Ministry required its application at the beginning of 1963, on a reduced scale, and in the public schools. The experiment involved five courses in as many schools of varying characteristics, though all in the federal district of Buenos Aires or nearby. The distribution aimed to assure efficient supervision and to determine the different responses made by students of different social and intellectual levels to the new demands. The teaching personnel was carefully selected from among those who had had scholarships to intensive courses for modernizing mathematical knowledge. Doctor Santalo, author of the intuitive geometry program for the first course, also wrote a guide for the proper implementation for the program. In addition, the personnel involved in the experiment met periodically, and they still do, to work out the best ways to handle the problems of content, method, and organization which always areise with this sort of innovation. Parallel to the implementation of the first course, a seminar was organized for discussing topics for the second course. This procedure was followed progressively every year until culminating recently in topics for the fifth course program which goes into operation in 1967.

Meanwhile, encouraged by the satisfactory results obtained, educational leadership authorized the expansion of the experiment so that now it involves 28 teachers functioning in 41 courses in 10 institutions in the capitol, the outskirts, and other parts of the country.

As the experiment proceeded and successive courses were given three annual reports were published and sent to all national secondary institutions so that all the mathematics teachers might have direct information about the areas of intended reform, the successes achieved, and the difficulties encountered.

The reports published are generally encouraging. For the ent they show that no teacher in the experiment - not with- nding the efforts required of him - is desirous of returning traditional teaching programs. With regard to the students, experiment has shown that they have an admirable capacity ace new situations, a sharp critical sense, a good discipline, ater interest in the subjects, and the feeling that mathematics as accesible and attractive at least as other subjects. They particularly enthused about the opportunity to participate e way in the creative process by not having to passively accept hematics as a fixed structure but rather using their ingenuity their skill to formulate observations, deduce consequences, and ve problems. The student is grasping knowledge by his own ivity rather than by a repetition of reasonings of others. experiment has shown, for example, that the fact that the first urse eliminates intuitive geometry of an axiomatic nature, and is ndamentally based on using the knowledge which the student quires by direct observation to develop reasoning and simple eductions, points to a way of teaching and learning which seems o correspond very well to mental characteristics of the 12 or 3 year olds. On the other hand, the requirements of later courses gradually familiarize the student with axiomatic reasoning and it appears that around the age of 15 the student admits the advantage of the method not because it is imposed on him but because he is personally convinced. The favorable evolution of the students in this regard has been obvious. By the age of 15, they are not satisfied with propositions based on the observation of individual cases; they ask for demonstrations of general validity, they formulate significant questions, reflect upon the answers, and show relative ease in using precise mathematical language.

Notwithstanding the advantages pointed out, the experiment has met with some problems. The principle one lies in the fact that five years of high school are not enough or the class hours program are insufficient for implementing the new program. When some topics are not covered in a particular course, they must be transferred to the next course, causing an extreme shortage of time there. The situation admits of four possible solutions: Reduce or readjust the experimental programs. It appears premature and perhaps inconvenient to transfer a good part of the intuitive geometry program to the last year of elementary level (which would weaken the probability of good instruction) at least for the present. Extend the high school courses to six years, a possibility that has already been discussed but which involved general educational policy which in Argentina is quite independent of the experiment in mathematics. Or increase the weekly class hours for the subject. This last is probably the most feasible, but would be related to plans for reform which also depend on future general policies.

In this regard I should point out the strong movement in Argentina in support of the intensification and modernization of instruction in all the basic sciences. This movement can only prove beneficial in mathematics and it is probable that a combination of adequate measures will permit the implementation of the normal experimental programs, especially once they have been adjusted from the viewpoint of general adoption and are supported by sufficient textbooks in Spanish and other needed conditions. Teachers of the experimental courses have pointed out the difficulties incurred by the fact that they and the students were not supplied in time with adequate texts in Spanish. Recently this inconvenience has been eliminated for the first three courses, with the appearance of textbooks which have been written by the experimental teachers themselves. The publication of new texts and working guides prepared outside of the faculty is emergent and this should shortly provide valuable assistance.

Modernization of In-Service Personnel

For their implementation, the new programs required specially equipped personnel recruited from among the in-service teachers. A compilation of data on all mathematics teachers in over 500 public high schools in the country resulted in the following table:

	Number	Percent
Total of Teachers included	2,800	
with teaching degree in their specialty	1,700	60%
with an applied degree (engineer, etc)	500	20%
with substitute degree (elementary school teacher)	440	16%
with no degree (1%) or with an irrelavent degree	110	4%

Notwithstanding the relatively encouraging picture of the statistics, Argentina, with some exceptions, suffered the same as other countries in that the in-service personnel, degrees notwithstanding, were not prepared to meet the requirements for insti tion in modern mathematics. For this reason another of the points agreed upon during the visit of Professor Stone in 1962 was put into operation, the organization of intensive summer courses for in-service teachers. The initiative came from the National Office of Scientific and Technical Research, and since then, with the support of the Council and the Ministry of Education and Justi and now also the Office of the Secretary of State for Cultural Education, five courses were organized each running six weeks. To date a total of 200 high school mathematics teachers have attended. From among them were chosen those who direct 41 experimental courses; and from this group also have come many of those who are spreading the reform ideas or heading working and study groups which organize courses or lecture series of a somewhat lower level in the interior of the country.

Along with the summer courses, other courses have been given during the school year, usually lasting two or three months; also workshops, lectures, seminars, and demonstration classes have been held. One can therefore affirm that about half of all the high school teachers in service, in public and private schools, have had the opportunity to attend at least one of the aforementioned kinds of classes in which topics of modern mathematics were discussed. Official informational circulars published in education reviews, and even the general press have given this matter a great deal of publicity, so there is no longer any doubt that everyone connected with mathematics instruction in the country is aware of the reform movement and its general characteristics. Everyone also knows or can find out easily about the bibliographical sources to which they might have recourse for more information.

The following table provides a clear picture of the present situation regarding the attendance at courses and lectures:

Total of high school mathematics teachers (2800 are in public school and 2200 in private)	5000
Teachers who attended intensive summer school courses organized by the National Council of Technical and Scientific Research	200 (4%)
Teachers who attended at least one course of no less than two to four class hours with final examination	400 (8%)
Teachers who attended at least one lecture series such as those given by E.Castelnuevo, L.Felix, or G.Papy without final examination	1000 (20%)
Teachers who attended separate lectures	2000 (40%) or more

No absolute statement can be made as to the number of high school teachers presently being conditioned to satisfactorily mee the requirements of an experimental course. The figure is estima at no more than 10% of the total number in service, that is, around 500. This means that the generalization of the experiment can only be accomplished in stages and also explains the caution exercised in incorporating modern mathematics topics into the traditional programs.

All methods must continue to be employed to improve in-servi personnel.

The Education of Future Teachers

One of the great objectives of the reform in mathematics instruction is to equip the future teacher so that he may continu to develop on his own as scientific and methodological advances require the updating of his knowledge once he has begun teaching The modernization of in-service teachers which is such a problem now because of the expense in time and money among many other things, will continue to be a problem unless some solution ti it discovered in the meanwhile. It therefore seems fundamental that w analyze the education of the future mathematics teacher in order equip him during his studies with the knowledge and ability that will enable him to successfully absorb the changes which are sure to come. The working document which we presented to this conference through Dr. Louis A. Santalo points out with some emphasis this essential characteristic, which is completemented and supplemented by the other conditions therein described: a strong mathematical education - extensive, intense, modern; a good pedagogical preparation, oriented toward the concrete problems of the professional world; adequate familiarity with some other science in which mathematics is instrumental; and finally, satisfactory general liberal arts background supported by some course in the history of mathematics and the evolution of scientific knowledge.

For a long time, teacher education courses for high school mathematics teachers in the Argentinian Republic included four years of study which may be followed independently or parallel and sometimes required, with the teaching program in physics. Until recently, content was traditional as far as the mathematics was concerned: three analyses courses (of which the first was really algebra), a course in geometry, one in metric geometry, one in projective geometry, one in descriptive geometry, two of analytic geometry, etc. And as far as teacher preparation went, the emphasis was on philosophy and general pedagogy although practice teaching was also important.

It is only since 1962 that we notice a change, with the introduction of courses in modern algebra, probability and statistics, and higher mathematics, all related to mathematics education. As far as teacher training goes, we note the availability of the course in child psychology, one in adolescent psychology, and one in psychology of learning. But today the reform movement has not been widespread; people are apparently waiting, perhaps with excessive caution. We may assume that the recommendations of this conference may in some degree contribute to the acceleration of the process and that the Argentinian Republic will soon have teacher training courses which meet current and future needs of mathematics instruction at the high school level.

HIGH SCHOOL MATHEMATICS TRAINING

H. Renato Volker
and
Luis A. Santalo

The extraordinary development of mathematics knowledge during the recent decades and the growing possibilities of its application in the various sciences and technologies necessitate revision of content and instructional methods in mathematics in the high school and consequently a change in teacher training for this level.

There is general concensus about the material of so-called modern mathematics to include in the high school program but at the same time the teachers now in the schools are not always equipped to do this, not having been trained to adapt themselves to changing situations. In addition, there is the need to adjust the training of future teachers of high schools to the new demands. This raises the question of the best way to accomplish reform which may be conditioned to the possibilities and characteristics of each individual country. This document seeks to outline aspects considered basic for this training and, without entering into details, characterize them so that they may be discussed, amplified, and made concrete as far as possible.

Basic Aspects

1. Mathematics Education

Competent teaching requires adequate scientific informatic in extension, intensity, and up-to-dateness. Hence, the high school teacher must possess solid and up-to-date knowledge considerably beyond the content which he proposes to transmit to his students. Only on the basis of mastery of his subject, with a critical sense and from a high level can the teacher be able to determine clearly the best form of and the degree of thoroughness with which it should be taught. The future high school teacher should receive his scientific training in the university or institutes of similar category; and there is no reason why the courses he takes, particularly in the first years or semesters, should be very much different in intensity and extension from those taken by future professional mathematicians A basic seminar at the level of his studies, which would equip him for independent handling and for improving his own formal education will be invaluable for his future development. On the other hand, in the advanced courses, it must be kept in mind that we are not preparing the scientific researcher but rather a teacher who certainly has to know thoroughly the various mathematics disciplines and their interrelations but at the same time needs particular preparation with regard to the adaptation of topics of interest, for possible teaching to high school students.

2. Pedagogical Training

Parallel to scientific preparation, but preferably in the advanced courses, it is necessary that the future teachers receive adequate pedagogical training on specific questions of mathematical instruction. General education courses tend to be based on philosphy courses and usually the latter are given at the beginning of the study program, that is before the candidate has either the information or the adequate scientific maturity for appreciating the needs for philosophical foundations. We assume that this has contributed in part to the lack of prestige of education courses among students of the sciences in general; but it may be assumed also that a reorganizing of these questions will produce a change, especially of educational training is oriented toward concrete problems which the high school teacher must face in his professional work - problems for which accurate solutions are vital to successful instruction. Thus, for example, individual and corrective behavior of students in the face of the need and the opportunity to learn is of interest to teaching. The matter is linked to adolescense psychology and group dynamics. It is also fundamental that the teacher know theory and techniques of learning to apply them to mathematics instruction, especially when new matters derived from the advanced levels are being adapted to high school levels in the clearest and most accessible form for students. There would seem to be justification for a special course or seminar dedicated to this kind of work especially if it could be also used to familiarize future teachers with the work and conclusions of meetings and congresses that have been held on this subject. This would also involve training them for critical creative and reform work directly linked with their daily duties. The period of practice teaching - lengthened or shortened according to particular aptitudes of the candidate would be an opportunity to try concrete application of acquired knowledge and techniques.

Aptitude for Change

Inasmuch as the great advances of mathematics as a science require rapid modernization of content and instructional methods at the high school level, the teacher of the subject must infuse his professional work with dynamics. That is, he must be equipped to handle change, which is a concept not developed in the traditional training of mathematics teachers. The lack of this outlook accounts for current difficulties with rapid adaptation of in-service personnel. Their modernization has to be carried out by all kinds of courses involving great cost of time and money. It would be prudent, hereafter, to provide for great flexibility in teacher training; that is, to so structure the major so as to assure aptitude for change. This means equipping the future high school teacher to continue study on his own once he has entered into professional activity and to consider it mandatory to keep up to date on scientific and methodological advances by means of reviews, specialized books, and systematic attendance at professional meetings. Perhaps this demand seems pretentious and difficult or impossible to meet, considering that the teacher is burdened and busy. Nonetheless, it is reasonable to assume that a seminar on topics of mathematics as a pure science and another on the adapting of modern mathematics questions to the high school is necessary. Similarly, some courses on fundamentals of mathematics, presented as an overview, will leave its mark on future teachers and equip them to confront the kinds of responsibilities detailed above with success.

Relation to Other Sciences

The importance of mathematics in other sciences such as physics and economics, in which they are absolutely vital, is so great that it should be alluded to in teaching. This is not merely to help some students acquire a clear and more complete concept of the importance of mathematics as a factor for culture and progress, but also to show the very broad field of mathematical application - mathematics as an indispensible tool of the mind. Therefore, it is appropriate that the high school of mathematics teachers be well acquainted with one of these applied fields and be able to demonstrate aspects of it to his students. It would be very useful, then, to incorporate into the study plan of the mathematics teacher some courses in mechanics or the application of mathematics to physics, economics, or biology, to cite only a few of the possible.

In some countries it is traditional for the teacher who teaches mathematics as his main subject to also give courses in physics and chemistry. This certainly requires that we incorporate into the study plan of the mathematics teacher a sufficient exposure of subjects in a minor specialization to guarantee professional competence. This concept of a double major may be useful in many places, especially in smaller schools and localities where it is difficult to find high school teachers for the basic sciences; but it cannot be recommended as a policy. Rather we wish to emphasize the need for the mathematics teacher to be sufficiently familiar with another field of knowledge to be able to use his mathematical knowledge as an instrument and to demonstrate to his students the use of mathematics in applied science. It clearly follows that this will require a substantial preparation in the second field and that this preparation could be the basis for obtaining the second teaching degree or in any case stronger professional qualifications.

Mathematics and General Culture

The high school mathematics teacher should bear in mind in his instruction, first, the specific content of his subject; second, as has been indicated, the possibilities of its application. But as an educator he cannot eliminate his duty to contribute to the general culture of this students. The notable influence that mathematical thought has had on almost all branches of knowledge and technology in this century burdens the mathematics teacher with a grave responsibility. He must be thoroughly familiar with this influence of mathematics and be abreast of the great stream of mathematical thought and its relation to general scientific evolution in the past. Hence, it seems advisable that the program of the high school mathematics teacher - preferably in the final year - include some course about the history of mathematics or evolution of scientific knowledge or mathematics and logic or similar matters.

Content

It is common practice to divide intermediate or secondary instruction into two phases: the basic or elementary phase (often the same for all students) and the higher phase in which various orientations are found - the preparatory diploma (liberal arts or science), the teaching profession, commercial instruction, and others.

Basically, then, two possibilities are evident:

a) training teachers for a single common level for both phases;
b) differentiation between the training of teachers for the basic phase and the training for those of the higher phase.

In the U.S.A., even more levels are differentiated among intermediate level teachers (see the recommendations for training of teachers of mathematics of the Mathematical Association of America, January, 1961), but for most of the other countries of America it seems that at the most two levels should be recommended.

In accordance with this criterian and keeping in mind the basic aspects dealt with in section 2, we list below a minimal aggregation of mathematical and pedagogical knowledge appropriate to the high school teacher, allowing for the possibility of two levels, as mentioned above, but without the descriptions of content which are modeled after the Dusseldorf Program, notwithstanding the fact that the latter was prepared with different objectives in mind. It should also be pointed out here that a teacher training plan prepared by the Department of Mathematics by the Faculty of Exact and Natural Sciences of the University of Buenos Aires plus the suggestions regarding it, formulated by Professors A. Pereira Gomes (Brazil) and C. Abuauad (Chile), were also born in mind. The order of the numeration and description is not meant to suggest in any way a study sequence or that the contents may not be regrouped perhaps in a more convenient way.

1. Analysis I. Real number; set of real numbers; limits; numerical series; functions of a real variable. Continuity. Derivatives. Maxima and minima of real functions. Successive derivatives. Taylor's formula. Reimann Integral. Primitive functions. Methods of Integration. Applications to areas and volumes. Analytic Geometry of the plane. Angle and distance. Polar coordinates. Some properties of the conics.

2. Analysis II. Vector algebra. Linear analytic geometry of space. Angle and distance. The quadratics by their reduced equations. Differential calculus of several variables. Taylor's formula. Maxima and minima of functions of several variables. Double and triple integrals. Vectorial analysis, gradiant, divergence, rotation; formulas for vectorial integrals.

3. Numerical Calculus. Systems of linear equations. Linear programming. Optimization. Numerical solution of equations. Differential equations. Approximate methods of integration. The possibilities of electronic computers. Some mathematical models of the social sciences, economics, or psychology.

4. Algebra. Sets. Relations. Functions. Relations of equivalence and order. Algebraic structures. Groups. Groups of transformations. Examples of abstract groups and groups of transformations. Sub-groups. Rings. Fields. Geometry of a finite field. Ring of polynomials with coefficients in a field. Division of polynomials. Greatest common divisor. Decomposition of rational fractions into simple elements. Fundamental theorem of algebra.

5. Linear Algebra. Vectorial Spaces. Linear independence. Bases of a space of finite dimensions. Duality. Linear mappings. Matrix calculation. Linear forms. Multilinear forms. Determinants. Proper vectors and proper forms of an

endomorphism. Characteristic equation. Reduction to a diagonal matrix. Quadratic forms. Euclidean vectorial spaces. Orthonormal bases. The orthogonal group.

6. Geometry. Metric, affine and projective transformations in a plane and in space. Examples of quadratic transformations. The Erlanger program. Concise notions of the Hilbert type of axioms for eculidean gèometry. Axiomatic geometry via linear algebra (Artin). Euclidean models of non-euclidean geometry. Elements of Combinatorial topology. Some problems of graphs in a plane. Topological classification of surface. Orientation and Euler's number.

7. Probability and Statistics. Axioms of the Calculus of probability. Some laws of probability: Binomial, Poisson, and Laplace-Gauss. Generative functions of moments. Sampling, estimation. Verification of hypotheses; χ^2 (Chi-squared); students' distribution. Statistical dependence. Correlation. Applications of statistics to problems of physics, biology, social sciences, and psychology. Evaluation of methods of teaching.

8. Foundations of Mathematics. Algebra of sets. Boolean Algebra. Symbolic Logic and propositional algebra. Examples of applying Boolean algebra to the theory of circuits. Axiomatic development of cardinal, integral, and rational numbers. Short history of the ideas and schools of mathematics.

9. Complement to Geometry.[1] Differential forms. Exterior differential calculus. Stokes formula for simple cases. Differential geometry of curves and surfaces of euclidean space. Geodesics. Total Curvature. Elements of geometry of Reimannian space. Tensorical calculus.

10. Complement to Analysis.[1] Analytic function of a complex variable. Conformal mapping. Examples of Reimann surfaces. Cauchy's integral. Residues. Calculus of variation; classical problems. Ideas of normed spaces. Examples.

11. Seminar in elements of mathematics.[1]

Observations

1. The subjects indicated with the sign (1) may be eliminated in the preparation of teachers for the basic phase.

2. The study plan should also have, in addition, some supplementary subjects as well as one or two courses in physics (principally mechanics or mathematics physics) and others of an educational nature (mathematics education, psychology of learning, etc.)

3. It might be interesting to include in the study plan a seminar on the teaching of modern mathematics with information and discussion about the experiments tried in various countries and the results and recommendations of international meetings on the subject.

Model Plans

By way of illustration the following study plans are group by year and the subjects referred to in the preceeding chapter are followed by an indication of the number of credit hours (weekly hours) for which they would be presumably taken.

First Year

Algebra	6 Semester Hours
Analysis I	6 Semester Hours
Pedagogy	4 Semester Hours
Psychology of adolescents	4 Semester Hours
	20

Second Year

Linear Algebra	5 Semester Hours
Analysis II	5 Semester Hours
Probability and Statistics	4 Semester Hours
Physics (Mechanics)	7 Semester Hours
Psychology of formation and dynamics of groups	3 Semester Hours
	24

Third Year

Geometry	5 Semester Hours
Complement of Analysis	4 Semester Hours
Numerical Analysis	4 Semester Hours
Seminar in Elements of Mathematics	5 Semester Hours
History of Contemporary Science	3 Semester Hours
Mathematical Didactics	3 Semester Hours
	24

Fourth Year

Foundations of Mathematics	4 Semester Hours
Complement to Geometry	4 Semester Hours
Mathematical Physics	6 Semester Hours
Seminar in Teaching Mathematics	4 Semester Hours
Practice Teaching	6 Semester Hours
	24

(Note: The total hours devoted to mathematics and mathematical education is 92.)

The plan shown can, of course, be developed also in semester courses - or quarters as they are called in some countries. In such cases the credit hours given to teach subject should be doubled and a sequence established so that no subject may be taken without the proper correlatives being passed first. That is, for example, subsequents to analysis could not be taken without prior passing of analysis I and analysis II.

PART III

MATHEMATICAL DEVELOPMENT

1961-1966

INTRODUCTION

The First Inter-American Conference on Mathematical Education was held in Bogota, Colombia, December, 1961. At the end of the conference, a number of recommendations for the improvement of mathematical education were made and directed to the ministries of public education, the universities and teacher training institutions, and to interested foundations. The following reports show the activity and progress, or lack thereof, in the various American States from 1961 to 1966.

The reports were not solicited from the governments or ministries of Public education. Rather, the participants from the various states to the Lima Conference were requested to gather and present this information in the best manner they could, acting within their own country. The reports are therefore informal, not official, and are to be regarded as the best estimates that the highly qualified reporters could present.

No attempt should be, or is, made to compare progress in one country with another. Each state has its unique organization of education, with its own problems, and its own manner of seeking improved mathematical education. These reports should be looked upon as a temporary springboard for the necessary steps toward improved education. In a few years, it is to be expected, that the ministry of public education in each country will appoint a qualified national commission on mathematics for which the duties will be to prepare a rather exact report of the status of mathematical education, the steps needed to improve the program, and means of accomplishing desired goals during the years ahead.

THE ARGENTINIAN REPUBLIC

Basis of the Reform Movement

The Republic of Argentina has responded to the tendency to renovate its mathematical instruction. The reformulation began at the University level, where the faculty of natural and physical sciences of the University of Buenos Aires was one of the first to initiate drastic reforms in its programs and methods, and thus becoming a nucleus of the reform movement in mathematics education in Argentina. The suggestions of international congresses, lectures and seminars were initiated by responsible people who were interested in this question. The results of the seminar of Royaumont (1959) were particularly decisive along with the Dubrovnik report (1961) and the recommendations of the first Inter-American Conference on mathematical education in Bogota (1961). The recommendations of Bogota provided direct impetus for the initial work of the reform movement in secondary education and continues to inspire changes in that area. Shortly after the Bogota conference, Professor Marshall H. Stone visited Buenos Aires and helped to formulate the following plan of action:

a) The formulation of a program of modern mathematics for high school to be applied on a small experimental scale;

b) the preparation of textbooks corresponding to the program;

c) review of the program and the text in the light of results obtained in the experimental courses;

d) the setting up of summer courses for high school teachers in-service in order to familiarize them with the new program; and texts;

e) obtaining the support of educational authorities to gradually set up the new programs in all the schools;

f) modernization of the training courses for high school teachers.

All of these proposals have been accomplished, or are in the process of being implemented.

The Experimental Program

A representative group of teachers from the mathematics department of the faculty of natural and exact sciences of the university of Buenos Aires formulated a new program in 1962 to be tried out in experimental high school classes. It was conceived as six courses, but does not lose its unity if the last course is omitted. This is a necessary provision since the standard Argentine high school following upon six grades of elementary school has only a five year sequence. It may be pointed out that high school studies are relatively short, in fact, among other difficulties, this constitutes an obstacle to the fullest development of mathematics programs.

The experimental program is characterized by a reduction in traditional Euclidean geometry and in trigonometry - which took up almost half of the available time in the normal courses. As a replacement, the study of vectors, linear transformations, matrices, inequations, linear programming, and matters of modern algebra (sets, relations, functions and structures) are introduced. Algebra, then, has replaced geometry, even for introducing the student to axiomatic reasoning, since it permits this in a simpler manner. The experimental program also includes topics of statistics and probability for the sake of interest and application.

The new program was introduced gradually, beginning in 1963, in a limited number of institutions in the national capital and cities in the interior. The teaching was put in the hands of selected personnel which had been exposed to orientation courses in which the working groups, year after year, agreed upon the best way to proceed. The experiment has now reached the fourth year. Each year a critical report of encouraging results was published. In general it can be said that the students are responding better and more enthusiastically to the demands and that the teachers have no desire to return to traditional instruction.

The success of the experimental programs caused the educational authorities to decide to include certain mathematical topics of modern mathematics in the traditional programs beginning in 1966. This refers to topics which are judged to be unpostponable and which provide conceptual unity and complement rather than replace the rest of the program. Topics of such a nature are relations, functions, inequations, vectors, statistics and probability. It was thought that in this way the ground work would be laid, particularly in the teaching personnel, for a generalization of the experimental reform in the future, when the program is stabilized and can be administered with adequately trained personnel.

Publication of Textbooks

At the outset the reform movement had to rely exclusively on foreign publications which were understood by teachers but not by students. One of our major preoccupations was to provide everyone with reference books, textbooks, working guides, and other material in Spanish to facilitate familiarization with the subject. At first these were only translations and short pieces of specific topics, but during recent years works have been appearing which correspond specifically to instructional needs. The following list is an incomplete but illustrative example:

L. Santalo, New Trends in Teaching Geometry[1]

O.E.C.D., Elementary Concepts of the Theory of Sets, Properties, and Relations[2]

Catlar, Y.R. de Sadosky, Introduction to Algebra; Concepts of Linear Algebra[3]

O. Varsavsky, Algebra for Secondary Schools[4]

L. Oubina, Introduction to the Theory of Sets[3]

J. Bosch, Introduction to Symbolic Logic[3]

L. Santalo, Mathematics in the Secondary School[1]

Gabba and Dalmasso, Modern Mathematics[4]

Hernandez, Rozo and Rabuffetti, Basic Concepts of Modern Mathematics[1]

V. de Tapia and De Martino, Notes on Intuitive Geometry[4]

J. Bosch and Trejo, Intermediate Modern Mathematics[4]

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[1]For Teachers; [2]High School Level; [3]Basic University Level;

journal called "Elements" has been published by Professors [?]nfi and Besio (Buenos Aires) since 1963. It appears three times yearly and each issue features articles and information on the teaching of modern mathematics abroad as well as in Argentina.

In addition to the indicated publications, leaflets, articles, and lectures on contemporary topics in mathematics are printed, some for teachers, others aimed at the high school or basic university level. At this writing there are also several textbooks ready for publication.

Additional Training for In-Service Teachers

The National Council of Scientific and Technical Research organized a Summer course in 1962 which met daily for 5 weeks and was attended by about 40 high school teachers. The subjects taught were, basically, parts of modern mathematics and secondarily, methodological questions. This was an attempt to prepare personnel to teach the experimental programs. In 1963 another course was given, in 1964 there were two courses, and in 1965 another. All in all, 200 educators from all over the country attended this type of course and upon returning to their respective places of residence propogated the new ideas. In 1965 the aforementioned council designated the most outstanding students of these courses to give regional courses for high school and lower level teachers. One hundred forty-two teachers were favored with these courses. Along with this work, the ministry of education has been organizing a fruitful series of courses and lectures since 1962 to disseminate the idea of reform in mathematics education as being urgently needed and to offer to interested parties the opportunity to come into direct contact with subjects of modern mathematics via high level and prestigeous teaching by Argentines and foreigners. The following resume with approximate figures provides a clear image of the situation:

Total of high school mathematics teachers in service: 5000 (of these 2,800 are in state public schools, 2,200 in private institutions);

Teachers who have attended some intensive course, supported by appropriate scholarships: 20 - that is, 4%

Teachers who have at least attended a regional type course of no less than 24 class hours: 400 - that is 8%

Teachers who attended at least a series of lectures without any final evaluatory examination such as given by Emma Castelnuevo, Lucienne Felix, George Papy, etc.,: 1,000 - that is 20%

Teachers who attended some scattered lectures: 2,000 or more.

Since the foregoing figures are not cumulative, that is, each category includes the previous , one can conclude that there are approximately 500 teachers who could currently be considered equipped to teach high school courses in the reformed mathematics, that is about 10% of the in-service personnel. The other 90% need to continue their education. This explains the prudence of educational authorities in deciding to include in the regular courses only those new subjects considered essential. Gradual amplification is envisioned.

The National Commission for the Teaching of Mathematics

Following the proposal of the National Council of Scientific and Technical Research, the then minister of education and justice designated in December 1964 the commission to study means of modernizing mathematics instruction along the lines indicated by the first conference on mathematics education in Bogota. The commission, made up of mathematicians, educators, and high level administrators, served until August 1966, proposing a series of measures regarding plans, programs, courses, lectures, publications, etc., which had an impressive influence on the evolution of the reform movement.

The Future

Given the accomplishments to date and the perspectives that are becoming clear, it seems appropriate to concentrate future efforts in the following areas:

a) Continuation and amplification of experimental courses - presently numbering 41 - in order to be able to evaluate results on broad bases. Adjustment of the programs, textbooks, and methods employed with the aim of making them part of all regular courses. Encouraging publication of new school texts, workbooks for students, and other useful instructional material.

b) Intensification of advanced scientific and methodological training of personnel through the organization of intensive vacation courses and regular courses during the school year - workshops, lecture series, seminars, and working groups; also the publication of books, leaflets and articles for teacher study and orientation.

c) Reform of the High School teacher training courses in order to assure sound and modern mathematics education; Pedagogical training for concrete objectives of professional duties; Particular aptitude for adjustment to new scientific and methodological needs in the future; Basic familiarity with other sciences in which mathematics is used; Satisfactory general liberal arts education.

BOLIVIA

Mathematics is taught in Bolivia at the following different levels: 1. Regular Education

a) Pre-school
b) Elementary School
c) High School
d) Technical-Professional
e) University

2. Adult Education
3. Special Rehabilitation Education
4. Extra Curricula and Cultural Enrichment Education

At each level mathematics is taught in accordance with the particular needs; but in no case has any kind of research been attempted to reveal these needs. Progress in mathematics teaching in Bolivia is limited by physical, financial and human resource factors which are essential for educational improvement.

The level of mathematics instruction at all levels is relatively inferior to that which has been observed in some Latim American countries and due to certain factors among which may be underscored:

1. the shortage of qualified personnel;
2. inadequate education of teachers;
3. the persistance of traditional methods;
4. the small prospects of social and economic betterment for teachers;

5. low salaries

6. instability

7. the little interest shown by educational authorities in the modernization of:

   a. the training of teachers

   b. programs of study

   c. the administrative side of education.

Under these conditions mathematics instructions in the past few years has remained static instead of improving. There has been some deterioration which has important and enormous implications for educational structures of the country.

Bearing in mind that the impact of mathematics in the twentieth century has been truly spectacular, it is hoped that in the coming years the interest of both teachers and institutions concerned with mathematics will increase to reach levels that will permit Bolivia to enter the research stage.

Changes in Instruction Plans

At the elementary and secondary levels there is some interest in introducing modern methods of instruction in mathematics but one of the major obstacles is the lack of mathematicians to instruct the inservice teachers.

Another important brake on progress is the lack of connection between the high school and the university which at present have no direct nor indirect relation to each the professors who could be a source of interrelation between the two levels constitute an independent element which does not adhere too closely to the universities nor give of their ability to the high school level.

At the university level the situation is not much better since mathematics is not considered an independent discipline with many applications in modern science, but rather merely as a tool subject.

In 1966 this situation has been changed at the University of La Paz by the creation of an institute in which mathematics can be studied as a major field. The Higher Institute of Basic Science - as the new institute is called - replaces the former Institute of Exact Sciences which was a subdivision of the faculty of Engineering. The undergraduate School of Engineering will now provide instruction at the scientific level and in addition lend assistance to the schools of engineering, economics, medicine, etc., thus centralizing the study of mathematics.

The institute, in addition, will prepare science teachers at the high school level which will permit the modernization of mathematics instruction at the high school level in a few years, since we shall then have university professors with modern background and more advanced education than we have now.

The Institute of Basic Sciences has involved not only university professors but professors at lower levels since it hopes to be an effective instrument for the achievement of goals which should have been obtained some time ago.

The contracting of highly qualified teaching personnel has been a serious problem for the Institute of Basic Sciences in attaining the intended level of its function. This inconvenience may be overcome next year, but meanwhile only basic mathematics has been taught, for the benefit of mathematicians, physicists, engineers, architects, etc.

Courses for In-Service Teachers (Retraining)

At the high school level the number of teachers with mathematics degrees is very low since it is difficult to attract potential candidates from among college graduates. They find the picture of professional careers in medicine, engineering, economics, etc., considerably more attractive.

This low level of interest in mathematics study in the normal schools can be seen in the number of graduates from the two teacher training schools in the country:

Year	La Paz Normal School	Sucre Normal School
1960	-	-
1961	4	4
1962	2	6
1963	3	4
1964	-	14
1965	3	11
1966	9	10

The annual number of graduates is so small that it doesn't cover the placement of teachers.

Generally, the shortage is covered by teaching interns from the schools of engineering and economics (advanced students) or relative incompetents whose only qualification is a B.A. degree. This gives an indication of the crisis in teaching.

In view of these conditions it will be necessary to solve the fundamental program: Teachers should be trained by the university, which would mean an immediate improvement in the social as well as education level of mathematics teachers, and this aspect would influence all other aspects of our education program.

At the university level, except for few exceptions, mathematics professors have specialized preparation in their discipline the senior professors are professionals with engineering and economics background and university work in applied mathematics. This obviously implies some stagnation at the teaching level which is causing the deterioration of the present situation.

It is hoped that the creation of the Institute of Basic Sciences will result not only in the preparation of good mathematics courses but also in the retraining or modernization of the knowledge of teachers in service, through inservice teaching training courses with average duration of six weeks. By this means, it would be possible to set up a mechanism of activities and constant enlargement which might result in interesting other teachers and other students.

To date no teacher training or teacher's improvement courses in mathematics have been given in the country.

A small number of educators did attend courses in Lima and Montevideo but have not been able to move beyond the traditional study patterns.

In the coming year, 1967, the university of San Andres is considering offering a course (in the school of engineering and economics) in improvement of mathematics for the high school level and for teachers of the elementary courses. The primary objective is to introduce in-service teachers to modern mathematics by courses in set theory, linear algebra, algebraic structures and analysis.

Major Recent Publications

Since the demands of the market are small, very few scholars decide to publish works. High cost and inadequate distribution of the book are also obstacles which hamper publication. Indeed not only are there almost no original publications; little instructional material is published.

Some texts have been published for high school level but they follow the traditional mathematics instruction.

No literature has appeared in the past 5 years dealing with mathematics in Bolivia, except for some applications in economics and engineering in the Reviews of the schools of these disciplines.

Principle Activities or Meetings

The mathematics circle is very small and limited to few people; it is difficult to set up group activities and/or meetings. At the University of San Andres, a center of mathematics study was formed in August and September to encourage and disseminate mathematics activity.

This is the second center of its kind, the first, the Henry Poincare center of mathematicians and physicists, having been formed several years ago, disappeared little by little, due to lack of interest. The center of mathematics studies has definite intentions and its directors believe that in one or two years it will achieve ...

Current Needs

From the foregoing description it can be deduced that mathematics instruction in Bolivia is limited. It will be necessary to undertake at once a full diagnosis of the present situation in order to be able, on the basis of the results, to plan the entire future of mathematics.

Parallel with this diagnosis and consequent planning it will be necessary to improve knowledge of in-service teachers through workshops, seminars, etc.

In order to achieve the goals indicated by economic and social plans for instruction in general, and in mathematics in particular, it will be necessary to restructure the entire educational system. One of the first steps toward improving the level of instruction is to review the present. Educational authorities have shown some interest in such an investigation.

While these investigations are being undertaken, it would be helpful if national and foreign bodies would offer seminars and/or workshops for the secondary level and also for professors of basic sciences of the university. Since we do not have mathematicians now interested in educational improvement, we should acquire the services of visiting professors for several years.

Another obstacle to advancement of mathematics in Bolivia is the absence of specialized libraries.

The exchange of teachers for brief periods would be another effective means of elevating the level of mathematics instruction in Bolivia. The level of mathematics instruction in Bolivia is among the lowest of all Latin American countries.

BRAZIL

Secondary School Mathematics

In this report we shall consider the following:

1. Mathematical Education Conferences in Brazil; 2. Updating of Mathematics high-school teachers; 3. Programs; 4. Elaboration of textbooks; 5. Experimental classes; 6. Plans to be accomplished.

1. MATHEMATICAL EDUCATION CONFERENCES IN BRAZIL

About 1950 there was in Brazil among Mathematics teachers a great dissatisfaction. The education was outdated, the programs were inflexible and the changes, which now and then occurred, ignored teacher consultation. Education Conferences were organized aiming at the meeting of Mathematics teachers from all over the country with the purpose of developing the patterns for a common working plan. The 1st National Conference on Mathematical Education took place in Salvador - Bahia - from the 4th to the 7th of September 1955 and was attended by 94 participants. This Conference achieved mainly the purpose of presenting the national position concerning Mathematics education. The following questions appeared

"Are our programs organized in order to fulfill the real needs of adolescents?

Under the present programs, do we effectively develop in scientific education a human and cultural value?

Are our educational methods really updated?"

Conclusion: Mathematics education should undergo a change.

In 1955, of the Commission Internationale de l'Enseignement des Mathematiques (C.I.E.M.) published "L'enseignement des Mathématiques" which in 1957, at Brazil's 2nd National Conference on Mathematical Education, raised the question: "Classical Mathematics or Modern Mathematics in the high-school programs?" However, the majority of Letters and Sciences University Schools in Brazil offered an essentially classical education and could not accept the reshaping of their programs to modern mathematics.

From 1957 to 1959, when the 3rd Brazilian Conference on Mathematical Education took place in Rio de Janeiro, little had been done to improve the situation. The majority of Brazilian teachers still did not know Modern Mathematics.

The 3rd Conference recommended that it be required from the Mathematical Departments of the Sciences and Letters University Schools all over the country, the giving of preparation courses in Modern Mathematics for high school teachers.

At this time and because of the insistence of Mathematics high-school teachers, several Study Groups, Centres and even Institutes were organized in the country, for updating teacher knowledge. Examples are Sao Paulo - Study Group on Mathematical Education founded on the 31st of October 1961 and the Mathematics and Physics Institute of the Bahia Federal University established in 1960.

The Institutes and study groups began assembling teams of high-school teachers, who could update their colleagues, recently graduated from the University Schools without good background as well as registered teachers who practice without having a university background. The Group of Sao Paulo, larger and better prepared, presented to the 4th Brazilian Conference on Mathematical Education, which took place in Belém - Pará State - July 1962, their first use of Modern Mathematics in high-school teaching.

At the time, other groups anticipated starting the teaching of Modern Mathematics in high-school.

The work at the Sao Paulo Group in Para aroused some criticism at the time, but was undoubtedly the example for those who were already prepared for the change; but lacked the courage to face it.

At the 4th Conference we prepared "Suggestions for a Mathematics modern program in high-school", which we shall discuss later. It was after the Conference of 1962, due not only to the results achieved at that conference, but also to the publication of the Royaumont Seminar Report in 1960, the "Un programme moderne de mathématiques pour l'enseignement secondaire" in 1961, and to the recommendations of the 1st Interamerican Conference on Mathematical Education; which took place in December 1961, in Bogotá (Colombia) - that a reform movement on Mathematical education occurred in Brazil. The climax came during the 5th Brazilian Conference on Mathematical Education, in Sao Jose dos Campos (Sao Paulo), January 1966, where the objective already accomplished in the country and methodological suggestions by foreign and Brazilian teachers were presented. This Conference assembled three hundred and fifty Brazilian participants and was attended by foreign authorities such as Professor Marshall Stone, from the University of Chicago, ...

...y, from the Free University of Brussels, Professor Helmuth ...ker, from the University of Buenos Aires.

The Sao Paulo Conference was also attended by numerous Mathematics Professors of Mathematics of the University, showing their ever increasing cooperation in this updating process of high-school teaching.

UPDATING OF THE MATHEMATICS HIGH-SCHOOL TEACHER

The education of high-school teachers in Brazil is still a task of Letters and Sciences University Schools, from which we have some teachers who are able to face modern Mathematics education, but still many graduates lack education to deal with Mathematics of the second half of the twentieth century.

Therefore our first task was updating teachers. The means used for such preparation were:

a) Intensive courses: such courses were planned and have been accomplished in some states of the country since 1962, during vacations (June, July or January/February). They are also divided and given throughout the school-year.

Meanwhile the first stage considered as basic education course has been given in different parts of the country; since January 1962. The subject established for this stage are: Introduction to Symbolic Logic, Introduction to Set Theory, Main Algebraic Structures and Modern Practices.

This first stage has been repeated for hundreds of Brazilian teachers; given at first in Sao Paulo and Bahia, as mentioned before, such courses are now taking place in Pernambuco, Paraiba and Ceara.

The 2nd stage, as far as we know, has been accomplished only in Sao Paulo. It includes Modern Algebra, Geometry (including Affine Geometry), Matrix Analysis, Probabilities, Differential and Integral Calculus.

b) Mathematical Weeks: Beside, the intensive courses, the Mathematical weeks or teachers' meetings - lasting a week - discuss Modern Mathematical subjects, aim at the updating of the teacher In the state of Rio the National Association of Mathematical Teachers and Researchers organized, in November 1964, the 1st Rio Week on Mathematical Studies and Education which took place in Niterei. In the same place a 2nd week has been planned for December 1966. In Porto Alegre (Rio Grande do Sul) a "Mathematical Week", promoted by the Mathematics Institute of Rio Grande do Sul, took place in 1964. In Sao Paulo, Letters and Sciences University Schools from the Capital and the interior have promoted weeks of this kind.

c) Mathematical Seminars: Numerous seminars have taken place hroughout the country, specially in what concerns modern handling f subjects in high-school level presented in foreign books.

d) Lectures: Aiming chiefly at convincing teachers of the ecessity of updating and of the advantages of Modern Mathematics everal lectures have been given by Brazilian and foreign high-chool teachers and professors. Professor Lucienne Felix for nstance visited the country in August 1965 and gave several lecures in Porto Alegre, Sao Paulo, Rio de Janeiro, Salvador and ecife.

PROGRAMS

The reshaping of Mathematical education in high-school level requires not only teachers well integrated in the spirit of Modern Mathematics; but also duely elaborated programs for such purpose. However, in a country like ours, radical changes are not advisable. What is to be hoped for is a restructuring of programs which enable the teacher by means of the most modern pedagogical techniques to achieve the purposes of Mathematical education, which at this stage, will undoubtedly be experimental. Thus the programs should be flexible and subject to periodical examinations until something definite can be settled.

It was with these purposes that the Sao Paulo Study Group on Mathematical Education presented in 1962 the following: "Minimal subjects for a modern program of Mathematics in junior and senior high-school."

a) Minimal Subjects for junior high-school:

1. Integers; fundamental operations; properties. Systems of enumeration.
2. Divisibility; multiples and factors; prime numbers.
3. Exponents and radicals; square root. 4. Fractions; fundamental operations properties; exponents and radicals. 5. Positive and negative numbers and zero; fundamental operations; properties.
6. Simple study of principals of plane and solid geometric figures; measurement of their lengths, areas and volumes. 7. Ratio and

proportion; applications. 8. Rational numbers; fundamental operations; properties. 9. Algebraic computation; polynomials whose coefficients are rational numbers; fundamental operations; properties. 10. Equations of the first degree on one unknown; inequalities of the first degree in one unknown; simultaneous inequalities 11. Algebraic fractions; fundamental operations; properties. 12. Functions; graphical representation of functions in a cartesian coordinate system. 13. System of two linear equations (2 unknowns); graphical interpretation. System of three linear equations (3 unknowns). 14. Systems of inequalities of the first degree in two unknowns; graphical interpretation. 15. Fundamental elements of plane geometry; point, line, half-line, segment, plane, half-plane angles, bissectors. 16.Polygons; generalities; the study of triangles. 17. Perpendicularity and parallelism in the plane; the study of quadrilaterals. 18. Circumference; properties; relative positions of line and circumference and of circumferences. 19. Irrational numbers and real numbers; fundamental operations; computations involving radicals. 20. The quadratic equation in one unknown; the quadratic function; equation and systems which can be reduced to the second degree. 21. Proportional segments; similar polygons; sine, cosine and tangent of an angle. 22. Metric relations within triangles. Sine and cosine laws. 23. Metric relations within circles; regular polygons. 24. Areas of polygons; measurement of the circumference and the area of a circle.

b) Minimal Subjects for senior high-school:

1. The function of the 2nd degree. The complete study of the quadratic function and applications. 2. Coordinates of a point of a circumference with center at the origin. Applications of the trigonometric, relations within triangles. 3. Identities; simple trigonometric equations and inequalities. 4. Introduction to solid Geometry; space and half-space. Parallelism and perpendicularity of lines and planes. 5. Dihedral, trihedral and polyhedral angles. 6. Polyhedrals: prisms, pyramids and frustum; geometric properties. 7. Round solids. 8. Point transformations: translation, rotation, symmetry and homothety. 9. The notion of sequence or succession of real numbers. Progressions. 10. The notion of power in the real field. Inverse operations. Logarithms. 11. Combinatorial Analysis and applications. 12. Elements of Plane Analystic Geometry. Equation of the straight line and equation of the circumference. Reduced equations, from conics. 13. Measurement of geometric solids. 14. System of linear equations; notion of matrices; applications. 15. Complex Numbers; fundamental operations; properties. 16. The study of polynomials. 17. Algebraic equations. 18. The notion of limit, continuity and derivative. Elements of the integral calculus; applications to the computation of areas and volumes.

Beside this work of the Sao Paulo Group, there is a modern program of Mathematics for high-school, elaborated in Bahia under the orientation of the Mathematics and Physics Institute of the Federal University of Bahia.

4. ELABORATING TEXTS

New programs require, of course, textbooks prepared to present them. It is clear that such books should be elaborated, for some time, in experimental character. Aiming at modern education many books have been published in Brazil for junior high-school. Usually the authors have tried to introduce some modern concepts, omitting others which are useless for the pupil. They are experiments which are trying to reach the target.

For senior high-school there is a publication by the I.B.E.C.C. which is an adaption to Brazilian programs and a translation of textbooks from the School Mathematics Study Group, S.M.S.G.

5. EXPERIMENTAL CLASSES

The inexperience of teachers, even of those who know the subject well and the use of experimental books, require careful treatment.

The supervision of experimental classes by pedagogues and :he statistical treatment of results will point out the changes :o be made.

As far as we know there are in Sao Paulo 75 experimental lasses and in Salvador (State of Bahia) 11 such classes in odern Mathematical Education. The experience in Sao Paulo began n 1961 and in Salvador in 1965.

. PLAN TO BE ACCOMPLISHED.

Although there is no national plan for the restructure of thematical high-school education in Brazil, some study Groups, Science Education Centers and Mathematics Institutes have decided to accomplish it following the program which includes the items mentioned below:

1. Education of teachers in the spirit of Modern Mathematics.
2. Restructure the programs.
3. Prepare textbooks for experimental programs.
4. Establish more experimental classes.

Higher Education

Mathematical education at Brazilian universities is divided into two levels: undergraduate (graduação) and graduate (pós-graduação). Previously to about 1930, training opportunities in mathematics were restricted to certain professional schools, such as engineering faculties, military academies, etc. It was only after that time that mathematical education, as such, began to be offered by Brazilian institutions.

1. UNDERGRADUATE STUDIES

The first school of mathematics in Brazil was the Faculdade de Filosofia, Ciencias e Letras da Universidade de São Paulo, created in 1934. It has been since then the best and largest undergraduate school in mathematics.

Other such faculties have been created in the major centers of the country afterwards. They are mostly federal institutions. A few of them are state or private. No trend towards outstanding education offered by private universities seems under way in Brazil, unfortunately.

Sound organized criticisms have been directed against existing undergraduate teaching and its relationship to secondary school education. Recently, undergraduate programs have been to a certain extent subject to revision, simplification and modernization in the major schools. The Ministerio da Educacão e Cultura seems have understood at last the importance of decentralization of control over such matters.

An effort to lower the level of the first-year university courses, so that they can match with the average training of entering students has been noticed. Whether such a practice is to be recommended to the leading schools which are expected to become centers of excellence is questionable.

GRADUATE STUDIES

A great deal of emphasis is being put right now on establishing organized graduate programs leading to the corresponding degrees: Master (Mestrado) and Doctor (Doutorado). Several government agencies and the Banco Nacional do Desenvolvimento Economico are seriously interested in furthering graduate studies in Brazilian universities, though mostly at the Master's level. Substantial financial support is becoming available to institutions that are capable of setting up Master's programs. Nothing really comparable is being done by such aid sources toward a Doctor's program.

In view of the diversity of tendencies and opinions concerning what a graduate program should be in the country, the Conselho Federal de Educacão passed a resolution at the end of 1965 defining in a remarkably flexible way what graduate activities are to look like; it was amended in 1966 to leave open the possibility of institutions to require Master's dissertations or not,, at their discretion. That resolution is based on the corresponding experience of the academic centers in Europe and the U.S.A. Fortunately enough, it is meant to guide Brazilian institutions, rather than impose on them a definite pattern reflecting only the thoughts of the legislators.

To a certain extent, mathematical training at the Master's level is being satisfactorily offered at the best centers, those in Rio de Janeiro and in São Paulo. It is questionable at the present moment whether further centers should be encouraged and supported to set up a Master's program. This is due to the fact that their undergraduate courses are not yet functioning well. There are not enough qualified students to enter the Master's program in such places. There is also a shortage of competent instructors at this level.

Mathematical training leading to the Doctor degree is being carried on mostly abroad, at good universities in the U.S.A., in France and in a few other countries. This aspect of graduate training of the young Brazilian students has been quite a success for many years, due to continued appropriate leadership. It is definitely, although slowly, as is natural, contributing a great deal to the improvement of mathematical education in Brazil at both the undergraduate and the graduate levels. Studying abroad at the best centers to get a Doctor degree should be one of the major goals of graduate training in Brazil for many years to come. Brazil has already, as a matter of fact, some, by international standard, good mathematicians, who can be faculty members at top universities. A few interesting doctoral dissertations have been prepared and passed in Brazil. However, if quality is to be an important feature of the Doctor degree, as it should, it is really too soon for the country to shift from the present fruitful arrangement to the doctoral training of students exclusively at the Brazilian universities.

The opportunities of doctoral work abroad stem mostly from an excellent fellowship program set up around 1951 by the Conselho Nacional de Pesquisas. To a lesser extent, they are provided also by CAPES, a Ministerio da Educacão e Cultura agency, and by other sources.

INSTITUTES OF MATHEMATICS

The first institute of mathematics in Brazil was the Instituto de Matematica Pura e Aplicada (IMPA), Rio de Janeiro, created in 1952 by the Conselho Nacional de Pesquisas. It has been since then the best graduate center, and has also established for itself the reputation of being the leading research center in mathematics of the country. IMPA has been very influential in Brazil as far as graduate education is concerned.

Brazilian universities are traditionally divided into faculties. Each Faculty has a Department of Mathematics of its own if mathematics is part of its course offering. In recent years, Brazilian universities became subject to new reform ideas concerning their structure, and started creating institutes of mathematics. At each university, the institute of mathematics is intended to subsume the whole of mathematical activities, whether undergraduate or graduate, teaching or research, pure or applied. At present, this is not yet actually the case. A certain conflict exists among the newly created institutes of mathematics and the already existing faculties. It is of course characteristic of a transition period. One can foresee that sooner or later the institutes of mathematics will take over.

The first university in the country to be created from the very start, according to the new outlook and reform conception, was the Universidade de Brasilia, a remarkable institution. This happened as recently as 1962. Its Instituto Central de Matematica has been entirely in charge of all mathematical activities within the university, has hired competent mathematicians and has functioned in a most promising way.

4. TRAINING OF INSTRUCTORS, MEETINGS, PUBLICATIONS AND LECTURERSHIPS

Most of the existing universities in Brazil offer poor undergraduate courses. This is due to the tremendous shortage of competent instructors, not only to the way universities are organized. However such universities are striving to improve by looking around for competent faculty members or by granting to their instructors leaves of absence with pay for them to improve their qualifications. Faculty members in these lesser developed academic centers have been encouraged to apply to CAPES's training fellowships in order to profit from the graduate programs offered by IMPA in Rio de Janeiro and the Universidade de São Paulo. This kind of activity ought to be greatly implemented.

The major meeting of mathematics is the Coloquio Brasileiro de Matematica, a series of all Brazilian mathematics colloquia held once every two years, since 1957, with the participation also of some foreign mathematicians. It is organized by IMPA. Each colloquium consists of several post-doctoral and graduate courses, several one-hour survey lectures, sessions for short research announcements, and panels on mathematics education. These colloquia have been held in Pocos de Caldas, Minas Gerais, with only one exception.

The Brazilian literature in mathematics at university level is still very poor when compared to the needs of the country. There is a tremendous deficiency of textbooks for undergraduate studies. Most of the publications for graduate studies have been put out by IMPA in Rio de Janeiro, by the Universidade de São Paulo, and by the Universidade de Pernambuco in Recife.

There is an excellent trend to increase lecturerships, both in the case of mathematicians moving from one Brazilian university to another in order to lecture, as well as in the case of foreign visitors coming to Brazil to that end.

CANADA

During the period under review there have been significant and widespread developments in mathematics programmes in Canada made possible, chiefly, because of the interest and enthusiasm of teachers in schools and universities. For example, a Summer Course in mathematics for Elementary School teachers in Ontario in 1966 attracted 1800 volunteers instead of the expected number of approximately 400: Many University professors and High School teachers have served on Curriculum Committees of the Department of Education in all the ten Provinces of Canada. Since education is a Provincial responsibility, the details of development vary across the Dominion. This report deals chiefly with the three most populous provinces: Ontario, Quebec, British Columbia.

1. CURRICULUM CHANGES - Changes in the mathematics curriculum have taken place in every province. In British Columbia a new programme was introduced in all schools in Grade 8 in the school year 1962/3. Year by year, up to the present, each grade has been revised to take care of the mathematics introduced the preceding year. Thus the new Grade XII course is in operation in B.C. during the 1966/7 school year. The three universities in British Columbia plan to adjust the freshman and sophomore mathematics courses for the students graduating from these new programmes. A similar revision of Grades I through VII was begun in 1961.

In Quebec, the Parent Report on Education (described below) recommended sweeping changes. 'Ten recommendations of the Commission pertain to the mathematics programme, including the proposal to teach the basic ideas of sets in Grade I, elementary mathematical logic in Grade IV, and to introduce High School students to the calculus of probability, statistical methods, linear algebra, differential and integral calculus.

In Ontario changes have been introduced in parallel at various levels of the school system. In 1961, without introducing significant change in the content of the official syllabus, the Department of Education suggested that set notation and basic ideas about the structure of the number systems be introduced into the Grade 9 Course. Because of the keen interest of teachers, in 1962/3 more than 50% of Grade 9 children were introduced to these notions. In 1964/5 new compulsory programmes were announced simultaneously for Grades 7 and Grade 11. New Programmes involving substantial changes are in effect in all schools of Ontario for the year 1966/7 in Grade 7, 8, 9, 11, 12. 13. The revision of Grade 10, which has been announced for 1967/8 will force a second, relatively minor, revision of the Grade 11-13 program beginning in the year 1968/9.

A feature of the new Grade 10 Curriculum, novel for Ontario, is a distinction between a Core (Real Numbers, Relations and Functions, Geometry) which all sutdents must cover, and four Optional Topics (History of Mathematics, Nature of Proof, Mechanical Aids to Computation, Algorithms), one or more of which will be taught according to the interest and background of the pupils and teacher.

The current curriculum in Grade 13 is divided between Analysis and Algebra. Analysis treats function, second degree relations and conic sections, the circular functions, transformations of the plane, including translations, rotations, reflections and dilatations, the classification of conics, sequences and limits, derivatives and their application to simple motion problems and easy differential equations. Algebra includes simple combinatorics, the binomial theorem for positive integral exponent, probability, vectors, equations of lines and planes, linear systems and matrices, linear transformations, complex numbers, polar coordinates, groups and algebraic structures.

In Ontario fourteen schools are testing, at Grade XII level, an experimental course on Algorithms and Computing. Material along this line will almost certainly be introduced into the regular mathematics sequence in the next few years.

2. RETRAINING TEACHERS - In British Columbia and Ontario the teacher's professional organization, known as the Teacher's Federation, is quite active in arranging short Summer Courses and In-Service Workshops to enable teachers to keep abreast of changes in the curriculum. Personnel to give leadership in these courses is provided by University Professors or by "master" teachers. Several of the larger local school systems have, in recent years, created a number of positions for "Mathematics Consultants", "Master Teachers" or "Mathematics Specialists" who are relieved of part or all regular teaching duties and are available for consultation by other teachers about pedagogical or subject content problems.

Many universities offer extensive programmes of Summer Courses for academic credit which teachers use to upgrade their qualificaons. These courses quite often deal with the new approach to school mathematics. No Canadian Universities have explicit plans for retraining professors.

PUBLICATIONS - Several publishing companies have produced a series of text-books, interpreting the new curriculum in Ontario from Grade 7 through 12.

The most significant document on education in Canada written during the past two or three decades is the Report of the Royal Commission of Inquiry on Education in the Province of Quebec, or the Parent Report as it is commonly known since the Commission was chaired by Monseignor A. M. Parent. This report, which contains over 400 recommendations, discusses every aspect of education under such headings as Contemporary Humanism and Education, Technical Instruction, Pre-University and Vocational Education, University Life, Teacher Training, Continuing Education, Exceptional Children, New Equilibrium of Programmes of Study, Music, Plastic Arts, Languages, Mathematics, Social Science, Physical Education, Moral and Religious Training Libraries. Quebec is a predominantly French-speaking Roman Catholic society which is reacting to the shock of the new technology. As such some of its experience is more relevant to Latin America than is that of the U.S.A. and English-speaking Canada. The three large volumes of the report, available in French and English, may be ordered from the Department of Education, Government of Quebec, Quebec City.

Professor E.B. Horne, of the Faculty of Education of the University of Victoria, in British Columbia, has prepared, as part of his Ph.D. thesis, a comprehensive report on "College Preparatory Mathematics Programs in Canada in 1964-65".

4. MOVEMENTS AND CONFERENCES - In some provinces there are organizations of mathematics teachers, such as the Ontario Association of Teachers of Mathematics. Three years ago the Canadian Teacher's Federation convened a nation-wide conference concerned chiefly with mathematics teaching in Elementary Schools and propose to call another Conference in the Spring of 1967 when the possibility of forming a Canadian association of mathematics teachers will be discussed. The Canadian Mathematical Congress, which unites mathematical scholars in the universities, has created a Standing Committee on Mathematics Education. This Committee on Mathematics Education will serve as a clearing-house for information and may well take steps to establish a common core of standards in mathematics instruction in order to facilitate the movement of students from one province to another.

5. FUTURE ACTION

(a) The traumatic experiences of recent years should teach us that, in the future, curriculum in schools and Universities should be kept under constant critical review.

(b) More efficient methods should be devised to enable teachers to keep up-to-date. School boards might use financial incentives to encourage this. Different variations on the idea of the "master teacher" mentioned above should be exploited. Teaching loads should be lightened with the explicit aim of encouraging teachers to read about and study recent developments in their field.

(c) A careful study should be made of the implications for the whole curriculum from Kindergarten to the Ph.D. level, of the variety of the increasing applications of mathematics to physical, engineering, biological, medical and other fields. In particular, the role of Computing Science as a subject of study in elementary and secondary schools and the advantages of an algorithmic approach to classic parts of mathematics should be explored.

CHILE

1. Background of some of the traditional problems of our intermediate level instruction in mathematics

Recent seminars and meetings have analysed the two fundamental areas of difficulty in mathematics instruction at the secondary level; the shortage of qualified faculty, and obsolete programs. These findings have been widely circulated. Some American groups ascribe the relative shortage of competent teaching personnel to the teacher training schools. This is a grave error. There is a logical chain, in which the first link is the low salaries of teachers of secondary mathematics. This produces a low enrollment in the mathematics departments of the teacher training institutions; those who do graduate barely fill the vacancies left by retirements. At the same time there is great pressure from the society to produce more teachers to cover the natural growth of the population and satisfy the increasing technological developments produced by the scientific flowering of our time.

Some academic institutions have responded with all they can lo: the acceptance of all applications for admission (compare this rith the general average of no more than 30% acceptance of pplications in the arts at the university) and generous promotions arrying out the wishes of the Department of Education. These easures have not sufficed because they are not creative. Society usually seeks its own solutions; early dropouts occur in the traditional teacher-training centers for the secondary level, and elementary level faculty begins to cover almost all the teaching duties of the beginning years of high school; institutions then arise for training mathematics teachers for the "early high school years," and in the outlying provinces new teacher training centers are set up which are either independent schools or departments of recently founded universities. In either case they lead a precarious existence in view of the shortage of qualified upper level university faculty in the basic sciences.

Despite these facts, the Ministry of Education carries on campaigns for the hurried completion of the curriculum of those who enter preparation for the teaching profession prematurely and to graduate them on the basis of three-week seminars. These are the reasons for the shortage of mathematics teachers in the high schools and for the irregular and inadequate training of a good number of them. The real solution for this situation is essentially political and therefore can not be accomplished by the university. Experience shows that traditional teacher-training institutes at the secondary level, while not anxious to modernize, have undertaken certain improvements in their mathematics instruction which are not yet recognized by the high schools.

The persistence in the high schools of a mathematics program which has remained unchanged for half a century has only intensified the problem. It has eliminated a vital part of every teacher's activity, namely the deeping up to date in methodological and scientific matters and brining to the Department of Education or the Institutes the fundamental criticism of the mathematics program in secondary teacher-training institutions.

The adoption of a modern, flexible program in the high schools requires no serious upsetting of present registration in teacher training institutes. It will pose problems for the Ministry of Education which specifically will have to re-educate teachers with ten or more years of service. It does present a lesser problem for the teacher training institutes, namely a logical organization of the study programs. Because of the quality of current faculty, every member feels responsible only for the course he is teaching. Since there is no collective functioning of the faculty - and efforts in that direction are resisted - each person sets up his own program with no regard for the program of his neighbor. This results in much repetition of subject matter and superficially in presentation. There is also a clear lack of natural connection between closely allied subjects.

2. Educational Activity in Chile in mathematics since the Bogota Conference

Progressive labor unions have suggested that any economic improvement must be linked to systematic advanced professional training, but their many attempts to set up state programs of advanced training, which would result in the desired economic betterment, have resulted in no more than the plans and agreements of the National Council on Education has never been put into practice. The establishment of a superintendent office for education in 1964 in charge of, among other things, the advanced professional training of teachers at both elementary and high school levels (with no practical effect in the secondary group) represented only a small vague legal contribution to these efforts.

It is worth noting that before the Bogota Conference the expression "advanced professional training" as it used to be used by the Ministry of Education and throughout the country with reference to intermediate mathematics teachers, had an entirely different meaning from that which it is now acquiring. The American mathematics group, SMSG, has also contributed to this awakening and has effected many aspects of mathematics education. The visit to Chile by a mathematician of international prestige, Marshall Stone, after the Bogota Conference in December 1961, has also helped. Professor Stone contacted the principal educational authority in Chile and published an article on mathematics education in intermediate instruction, which was duly circulated nationally and internationally. This three-pronged emphasis - dissemination of the Bogota Conference Report, the personal contact between Professor Stone and Chilean educational authorities, and the work of the SMSG group has resulted in some steps and attempts at improvement within the last few years and this in an atmosphere that has always been conservative and fearful of change.

A significant occurrence which showed the impact of the work described above was brought about by conservative actions of the Department of Education of the University of Chile in 1962. They quietly formulated a "new mathematics program for intermediate instruction." This was formulated and legally approved in a very short time, where normally such matters involve reports and more reports. The program, however, is not worth describing.

The secondary level mathematics professors union which until then had been principally concerned with economic gains, organized mathematics seminars in 1963-64 with weekly meetings and the help of some university professors.

The state technical university has organized two summer nstitutes (January 1965, 1966), for the improvement of mathe-atics teachers. They have been attended by small groups of high chool teachers. Thirty-nine enrolled in 1965 and 107 in 1966. rofessor Burton Jones was invited in 1966 and gave two lectures t the institute. Some of the participants have undertaken isolated experiments in high schools at Antofagasta, Santiago and Concepcion. Funds have been lacking to finance further attempts.

In April 1965 the Commission for Overall Educational Planning held a seminar in the Federico Santa Maria Technical University with the cooperation of the Ford Foundation. The Mathematics Commission accepted the general lines of the recent international conferences on mathematical education.

The Office of Administration of High School Education organized mathematics seminars in 1966 in the provinces, with the collaboration of universities and university centers.

3. The State Program of Systematic Improvement

With decree 17177 of October 23, 1964, the government started a National Improvement Program which is now considered an essential part of teacher training (at the elementary and secondary levels) rather than merely supplementary or irregular activity. The decree provides that the Superintendent of Public Education, the executive of this program, shall propose to the Minister of Education prior to the first of August of each year, a plan of activities for the improvement program for the following year. It further provides that the Minister shall decide on the proposal before the first of November. It enables the superintendent's office to solicit the assistance of the University of Chile and of other persons and institutions within or outside the country should it be necessary. This systematic plan of improvement, which has the financial support of the Ford Foundation, consults with a technical council of directors made up of representatives of the universities, the Ministry of Education, and the Teachers Unions. The council is supposed to provide technical orientation (although in practice it has functioned as a watchful body fearful that technical aspects might be developed beyond their own convictions). There are also a coordinator and executive secretary in charge of the administration of the plan and various committees of experts in close contact with the administrators and in charge of the technical work. This includes the preparatic of course programs, the production of study materials, the selection of professors and assistants for giving the courses and even the selection of students by competition for academic standing.

The improvement plan has provisions for adequate building with classrooms, library, laboratories, and dormitories for participating students. The administrators of this program hope that within 5 or 6 years, at the rate of 300 students per trimester, nearly all the secondary teachers will have been able to take advanced studies.

To understand more of the nature of this improvement program it is useful to point out that the old division of school instruction into 6 years of primary and 6 years of high school and liberal arts is now divided into two phases, one of eight grades which make up the elementary school and the other which extends from the 9th grade to the 12th grade and is considered the high school.

4. Initial Experience

The improvement program began this year with all the difficulties inherent in the lack of an appropriate place for the administrative personnel (the least serious of the problem), the lack of an appropriate place for theory classes, the lack of proper laboratories and the lack of dormitories for participants coming from a distance.

While the mathematics committee was busy with the formulation of the first programs for the improvement courses of January 1966, it was surprised to receive "the transitional mathematics program for the 7th grade" formulated in the Office of the Superintendent of Education without any consultation with the committee. The plan sought to organize methodological courses in laboratories to cover the transitional program and was intented for 107 7th grade teachers selected from a large group of interested persons. The committee pointed out the inconvenience of improvement courses for teachers of a particular grade, it suggested that it was desirable not to proceed along such lines in the future, and that it was desirable to have official contact with the technical office of the superintendent which is the executive arm of the transition program. It requested direct participation in the formulation of mathematics programs of intermediate level, which was the final objective of the improvement program.

This request for direct participation in the elaboration of high school mathematics programs, which seem natural to the committee by virtue of the character of the work it was supposed to do, has been reiterated a number of times to the administrative authorities. Although the coordinator and executive secretary shared the feeling of the members of the committee, they recognized that there was a small legal impediment, one which put the Superintendent of Education in charge of all reforms and attempts at reform of the mathematics program at the intermediate level. In the final analysis, this impediment is not a real obstacle, since the Superintendent is in power to consult persons and institutions outside his office if he so wishes. If the Superintendent acts with real disinterest, aware of the importance of the step involved, without unnecessary administrative jealousy and without assuming that outside activity is a matter of interference with his work, he can obtain a fruitful joining of forces. Some achievements have been magnificent in other subjects, in the joint hands of the committee of experts and the technical office of the Superintendent.

Of the 117 seventh-grade teachers enrolled in the March session about 110 were elementary teachers (without university degrees). This was a new situation which the committee had not forseen and for which it was not prepared. Through the kindness of Professor H. Fehr (Teachers College, Columbia University and executive secretary of the IACME) the mathematics committee acquired the collaboration of Dr. Bruce Vogeli who actively participated in the formulation of the program for the methods courses and laboratories which were offered. The courses began in the middle of March of this year and were given by the best professors in Chile, since it was felt that the entire future of improvement was at stake. The technical council of directors functioned as a critical and guiding organism. These courses, which were brief and constantly readjusted while in progress, provided a magnificent experience of the administrators and the teachers. However, with the exception of 3 or 4 participants, none of the 117 students were willing to sign their name to the only written test given at the end of the courses arguing that a poor examination might be used against a teaching career by the Ministry of Education.

The improvement program for intermediate level mathematics teachers began in July of this year, with the participation of 37 high school teachers selected from a group of 70 applicants The principle offerings were two courses, each of 30 class hours, one in algebra, one in geometry. The first covered the fundamenta structures of algebra including the concepts of vectorial space, sub-space, quotient space, isomorphism, bases, dimensions, metric and linear mappings and range of linear mappings. The geometry course was essentially the axiomatic system of Hilbert. The July session developed normally, included two written tests and was considered a great success by the participants and the committee.

COLOMBIA

1. Program Changes

a) No fundamental changes have been effected in high school instruction; current programs are more or less those of five years ago, although attempts have been made to modernize the program. The main difficulties encountered in restructuring mathematics instruction at the high school level have been the shortage of teachers with good mathematics and pedagogical backgrounds and the explosion of the high school student population.

At present, some textbooks on mathematics and its methodology for the elementary and high school levels are published in Colombia, They represent great progress compared to previous texts.

b) At the university level there are no mathematics programs in Colombia which might serve as models for all the country's universities, for instruction in the various faculties and in institutes.

Improvement of Teachers at All Levels

In recent years both regular and sporadic courses have been set up in Colombia for training teachers at the high school level. For example, the Department of the National University, in collaboration with the Colombian Mathematics Society has undertaken the project of preparing the training courses for teachers at all levels. This plan was undertaken in November of 1965; and about 400 students and lower level teachers have been taught the basic areas of mathematics and corresponding methodology. In addition, it may be pointed out that by passing these courses and fulfilling other requirements such as seniority, teachers may become enrolled in the national roster, which provides stability of employment and better economic conditions. Similar programs are in force in other Colombian Universities.

As yet the shortage of human resources has made it impossible to undertake regular programs for preparing university teachers. They have had to keep up to date only through lectures, seminars, and workshops. The Department of Mathematics of the National University has been developing a program of training of university mathematics teachers in Colombia. For the last two years they have instituted the major in mathematics at a fairly high level. This year two more universities are starting professional majors in mathematics.

Publications

Currently admirable efforts are being made in Colombia with respect to the publication of textbooks for university and high school instruction.

In addition, the Colombian Mathematics Review is published periodically and provides a form of general information and scholarly articles.

Meetings on Mathematics Education in Colombia

The National University of Colombia and the Colombian Mathematics Society have held the following conferences:

1963 - National Congress on Mathematics, Bogota, Oct. 9-11

1964 - National Congress on Mathematics, Medellin, Oct. 13-15.

1965 - Bolivian Congress in conjunction with National Congress of Mathematicians, Bogota, September 21-24.

1966 - Fifth National Congress of Mathematicians, Bucaramanga, September 29 - October 1

1966 - Meeting of the Deans of Faculty of Sciences and of Education on the theme "Unification of Mathematics Programs," Bogota

1966 - Meeting of the Deans of the Faculties of Economics, April 22-23, Bogota.

Needs

Perhaps what is most urgently and vitally needed in the country is the education of university professors and the creation of a mathematics society in Colombia. We believe it to be of fundamental importance for the development of mathematics to have high level teachers of mathematics.

We submit the following needs for the consideration:

1. That all countries make efforts to create a propitious environment for research and instruction in mathematics;
2. That Latim Americans solve the serious problem of the immigration of highly qualified personnel;
3. That there already exist a nucleus of mathematicians carrying on research which can be used in united effort for the creation of a research and study environment in all South America;

4. That it is necessary to break the traditional isolation of Latim American Universities;

5. That it is necessary to have an International Institute for the primary purpose of training and updating university faculty in mathematics.

We recommend the creation of a high level mathematics center in some Latin-American country with international support. The primary aims of this center would be:

a) the encouragement and development of research on the field of mathematics

b) the establishment of post-graduate courses for teachers and graduate students

c) the formation of a mathematics environment in Latin America to avoid the exodus of scientists from these countries

d) the acquisition of visiting professors for Latin American universities.

COSTA RICA

In the past five years the lower levels of university instruction have undergone changes that are of significance to the country.

The Department of Physics and Mathematics of the Division of Sciences and Liberal Arts has a dual mission: one, to fullfill the mathematics part of the academic preparation of students in the faculty of sciences and liberal arts (B.A., teaching degrees, advanced graduate degrees); two, to provide the mathematics needed by students from the technical schools (civil and electrical engineering, chemistry, economics, microbiology, odontology, agronomy, biology).

To meet the demands of this mission, the department originally organized its courses along lines proved inadequate by experience. This consisted of a common core of courses for the several careers, extending over a period of four years and for other students totally autonomous courses. For example, the courses in mathematics for a teaching degree in physics and mathematics, for a bachelor's degree in mathematics, or for higher graduate degrees in physics and mathematics, were exactly the same. This led to unsatisfactory results, since those who majored in mathematics received inadequate preparation because the courses had also to serve the interest of physics and education students. The latter in turn also felt a lack of the kind of mathematical study needed for their field.

In 1965 the department attempted to improve the situation by formulating flexible study programs to meet the needs of the separate majors. At the same time, the low academic level of the students entering the university was taken into account.

In this new plan there are common courses for departmental majors. The science majors take the same first year course but have a different sequence from the second year on.

On the other hand, there is a tendency to eliminate as much as possible the courses which the department gives for other schools. We have managed, for example, to assimilate into the regular departmental calculus courses several courses in calculus which used to be given for the engineering school.

The committee for the advisement of these curriculum plans was made up of distinguished authorities in the field who were also well acquainted with local conditions in our country, among them: Dr. Burton Jones, Dr. Horten Ninnott, Dr. Emilo Santos, Dr. John Ray.

Once the problems of administrative and academic organization of the university department had been solved, we faced the question of secondary school teacher training.

The demand for teachers at this level is enormous and the university has been unable to meet it. The following reasons have contributed to this situation: a) lack of professional prestige in the teaching profession, b) low salaries, c) academic difficulty of the subject which produces drop-outs; and a strong tendency on the part of those who do master the program to transfer to technical schools.

Some of these problems were mentioned in the Bogota Conference and efforts are being made in Costa Rica to improve this situation. To attack this problem it was decided to simplify the study program for teacher preparation, as a temporary measure, which while questionable from an academic point of view, was realistic. The present lack of preparation of the majority of teachers at the lower secondary levels is a result. It was thought preferable, however, to have teachers with some academic training rather than having teachers who have had no contact with the university.

Beginning in 1967, to encourage students toward a career in teaching, the university has offered a scholarship plan for those who wish to be teachers.

Secondary Level Teaching

In 1964, reforms were introduced in all subjects at the secondary level, including mathematics. It was possible to accompany this reform of the whole secondary system in Costa Rica with reform in the content of mathematics, following the example of other countries.

The magnitude of the project and the paucity of resources led us to expect serious problems from the outset. Nonetheless, results to date have been favorable in one respect through unsatisfactory in another.

The main problem faced by the reformers was the inadequacy of teachers. Attempts to solve this problem have not been very successful. Textbooks were published and massive summer seminars for teachers were organized to improve the mathematics education of the teachers. Results have been somewhat negative; it was observed that there was too much material and too little time, considering the inadequate academic background of the participants. These seminars were given during the summers of 1964 and 1965.

A commission was appointed to work with and evaluate (on a scale) the teaching personnel of the lower levels. This commission will formulate class lessons, for secondary school teachers, including the specification of teaching materials for students and for teachers. The subject matter will not be formulated by the members of the commission alone; collaboration will also be sought from university professors and from a select group of high school teachers.

During the months of January and February, the members of the commission will disperse to various parts of the country to form groups of 15 to 20 teachers and explain the topics. They will choose from within these groups individuals who will serve as a liaison between the members of the commission and the greater body of the teaching profession. At the beginning of the school year these liaison teachers will travel to the university to update themselves on the materials being developed. They then return to update the teachers in their area. In this way it is hoped to train the teachers in the discipline they teach.

DOMINICAN REPUBLIC

Under the plan of improvement for in-service teachers, a seminar of exchange of educational experiences was held in the Summer of 1966 for teachers of the National Educational Association of the U.S. and Dominican teachers. Teachers were from the elementary level and normal schools.

One of the areas touched on by the seminar was mathematics. The dominant frame of reference was modern mathematics. Instructional materials were produced and demonstrations were given.

Another improvement plan for in-service teachers which is presently being operated by the Bureau of Education is the Eastern Private Plan. This plan applies to rural teachers whose background is at the level of the 8th grade. It utilizes the assistance of volunteer teachers from the Peace Corps.

One of the areas of training is mathematics. The programs follow orientation to modern mathematics. Instructional material is produced and demonstration classes are given. The use of film strips and moving pictures prepared by the Latin American Institute of Educational Sinematography (ILCE) of Mexico on modern mathematics has proved very useful.

Presently there are plans for the training of high school level teachers in modern instructional techniques for mathematics as well as other areas. This project is included in the AID-University of San Jose, California-Dominican Government Plan for Technical Cooperation. The project described above includes theoretical and practical classes. Besides, in the Dominican Republic there is the report of activities by the Mathematics Department of Autonomist University of Santo Domingo (UASD).

The constant evaluation of mathematics education as well as the needs of technology and research have obliged the Autonomist University of Santo Domingo to undertake a combination of measures as part of a general plan to place itself in the contemporary situation ejoyed by the most advanced institutions of learning of its kind.

The immediate results of our preoccupation has been the creation of the Department of Mathematics, improvement courses for university mathematics teachers, improvement courses for teachers of the basic sciences, and a new common mathematics course for all students enrolling in the university.

For next year, varous new courses are planned, dependent upon international cooperation and our own efforts. Of immediate urgency are:

a) modern mathematics courses for university teachers;

b) modern mathematics courses for high school teachers.

In addition a new study plan is currently being formulated to enable us to offer a full masters program in mathematics by the end of 1967. This will make it possible to produce the university and high school teachers who are competent in mathematics rather than self-taught (with clear competence in a given technical area but not really mathematicians).

The Mathematics Department

For many years the mathematics education received by university students in the various schools had no central control which could orient the goals. This produced an inertia with respect to the evaluation of mathematics education and its future service to technical and research needs. In view of this, the Mathematics Department of Autonomist University of Santo Domingo was created by joining efforts to achieve the many advantages of departmentalization as well as favorable budget which is a real necessity for relatively small universities.

Courses for University Professors

As long as there was no mathematics major in our country, our university professors were generally Dominicans who had studied abroad or who had been graduated in some technical field. This has resulted in a lack of true mathematics professors. There was need, then, to train these professors in the new views of mathematics education as well as the indispensible knowledge for carrying out their functions.

Along these lines, the following has been achieved to date:

A course in modern mathematics which included propositional algebra andnaive set theory. This course was given by Professor Tebas Peyro. At the same time, to help raise the scientific level of the university, a course is presently being given in basic sciences for university teachers. This course is in mathematical analysis and includes: strucuture of the real numbers, the theory of equations, and integral and differential calculus of one variable. This course is given by the Mathematics Department through a team of teachers.

Basic Course for Students Enrolling in the University

The traditional inadequacy of students beginning university work has made necessary the creation of a minimum course in mathematics along the lines of pre-university courses given in other universities and which are obligatory for all students before beginning their major.

Eminent Project

We expect to implement the plans described above by the end of 1967. We shall very much appreciate every assistance from the Inter-American Conference on Mathematics Education and its illustrious members.

EQUADOR

In the last few years Ecuador has undertaken to reform the teaching of mathematics in secondary schools. Syllabi and programs have been modified in order to extend mathematical study by eliminating superfluous elements of the traditional program and textbooks, and to aid the student not only to manipulate but also to do creative thinking. As early as 1956, Dr. Antonio Rodriguez San Juan, technical expert for UNESCO, aided by a group of academic specialists, formulated a new mathematics program for all secondary school courses. These programs were gradually introduced around 1960 and are currently in use. While the new program still has a traditional appearance, it introduces vital changes in the preparatory program, particularly in the fourth and sixth-level courses for the physics-mathematics majors, where a) Introduction to Trigonometry, and b) Probability Calculus are respectivly prescribed.

In 1962, the disquiet of some mathematical scholars was abated by developing new programs for the secondary school based on modern mathematics. Mathematics teaching was then separated into two levels: a) the basic cycle, and b) the diversified cycle.

The first cycle includes course levels 1, 2, 3; the second, levels 4, 5, and 6. This separation into levels has been called "the Educational Reform."

To try out the basic phase of the educational reform eight "pilot schools" were selected to begin the new curriculum in the school year 1963-1964. Since then, the educational reform program for first, second and third levels has been developed. These first three levels constitute a common program for all pupils. The fourth year is the first level of the diversified cycle, or the preparatory course in Modern Liberal Arts, where students may major in physics and mathematics.

In these Liberal Arts courses, modern mathematics will be studied with a view to the joint treatment of the various branches of mathematics at each respective level. Hence, studies will include: plane and solid geometry, equations and inequations (simultaneously from the first level on); terminology and operations pertaining to sets will be introduced systematically and early; graphic representation of the elements of analytic geometry will be studied from the first level on. For the understanding of decimal numeration the teaching of various notational systems is required. Probability, calculus, and statistics will be treated.

The fifth and sixth level programs will be formulated during the school year 1966-1967, extending the use of modern mathematics.

While the educational reforms are being tried out in pilot classes, the current teaching of mathematics continues with the old programs. It should be pointed out that for the structuring and correction of the educational reform programs, the recommendations of recent International Conferences on Mathematics Education have been consulted.

Teacher Training

Seminars given by experts have been used to train faculties in the utilization of the new programs: In March 1966, Dr. Dederich gave a course in Modern Mathematics in the Colegio Nacional 24 de Mayo. It was attended by mathematics teachers of the capital. In August 1966, a course in Professional Development for Mathematics Teachers was given at the Central University. The seminar was given by teachers of the Ecuadorian Mathematics Society and dealt with the new tendencies in modern mathematics. In October 1966, a course was given in which a study was made of the contents of first-year modern mathematics courses, with methodology and the formulation of instructional materials. It is hoped that the educational reform will be fully operative beginning with the school year 1967-1968.

Suggestions

The study of modern mathematics has stirred considerable interest among teachers in Ecuador, but specialized consultation is needed to impart specific knowledge about the new approach at all the levels - primary, secondary, and university.

It would be very helpful if experts in modern mathematics could provide adequate sources for making known and putting into practice the new approach.

HAITI

In Haiti, the majority of mathematics teachers have been aware of the evolution taking place in the teaching of this discipline. The work and writings of Professor Lucienne Felix, Patrick Suppes, George Papy, Andre Revuz, have aroused much interest among Haitian educators and have helped them to improve the effectiveness of instruction. At present studies are under way for the proposal regarding the preparation of an official program based on new teaching methods in mathematics for high schools and the university.

Some private secondary schools have already begun to introduce modern mathematics into their programs. From the pedagogical point of view the results of this experience appear to be encouraging up to now.

After about twenty years, statistics has been included in the program of the law school at Port Au Prince and the Ethnology school. Two years later a course in statistics and mathematics was added to the program of the medical school of Haiti. Nonetheless, the teaching of mathematics in Haiti faces two major difficulties: a) a scarcity of qualified teachers; b) the problem of re-training present teachers in the modifications introduced by modern mathematics.

With the help of the French Institute, the Normal School of Haiti now prepares teachers to teach mathematics in the secondary school. An average of three teachers graduate from this school every year, which is truly inadequate in view of the high rate of population growth in the Haitian school system. This is supplemented by calling upon graduate assistance from the Polytechnic School of Haiti and the French and American Technical schools, or, if needed, upon established technical people known to be competent in mathematics.

Teacher Improvement

A number of mathematics teachers have already received scholarships to take advanced specialized courses in France, Belgium and the U.S.A. Some of them, upon their return, have voluntarily helped their Haitian colleagues to adapt themselves to the spirit of modern mathematics. It must be pointed out however, that we are still far from secure in the future of mathematics education in Haiti. It would be well to find adequate means for stimulating more careers in the teaching of this discipline, so that mathematics, which plays such an important part in the culture and technology of man's modern world, may properly assume its vital role in the intellectual formation of Haitian youth.

JAMAICA

The educational system in Jamaica is based very largely on the current system in England. This is true at both the High School and University levels. Pupils graduating from Secondary Schools (roughly equivalent to American High Schools) do so usually by taking the Cambridge examinations in a variety of subjects, at one of two levels, "O" (ordinary) or "A" (advanced). The requirements for entry to a course at a University (specifically, the University of the West Indies) will be the attainment of several A-levels, and this represents an educational level on a par with that acquired after one year at a University in the United States. Thus the system is such that the equivalent of the freshman year at an American system is covered in the secondary school - at least in the better schools. Insofar as not all schools in the West Indies are able to bring their pupils up to this level, and for other reasons, "preliminary" courses in most subjects are offered at the University. These are one-year courses which precede entry to the degree courses proper, and a student entering such a course is in approximately the same position as a U.S. freshman.

The courses at the University level are also organized differently from those at American Universities. Students are expected to follow a course lasting usually three years, at the end of which the degree is awarded (or not!). There is no provision for taking a course piece-meal, as is provided by the American credit system, and it is only in exceptional circumstances that a degree course can be dropped and resumed later.

These important differences between the educational system in the West Indies and that prevailing in the United States and elsewhere in the Western hemisphere need to be borne in mind in assessing the present status of mathematical education in the West Indian islands.

School Education.

The biggest problem facing the schools in Jamaica is a severe shortage of trained teachers. This is true of all subjects but perhaps more so in Mathematics than in some others. As a result of the shortage of locally-trained teachers schools are forced to depend heavily on inadequately trained personnel, or on teachers recruited from abroad. The dangers inherent in employing unqualified staff are obvious; the main trouble with employing teachers from abroad is that although these teachers are often very good, their appointment is usually only for a year or two. In consequence there tends to be a rapid turn-over of teachers, and a pupil may well find himself with a different mathematics teacher each year, or even changing teachers in mid-term. The effect of this in a subject like mathematics, in which continuity of presentation is all-important, is often to stultify the work of the good teachers that the pupil may from time to time have had.

The seriousness of this situation is well recognized by the educational authorities in Jamaica, and by the Government. Attemp have been made to improve it, but as yet there has not been any great success. In this sort of environment only a few schools have the opportunity to try anything radically new in teaching methods, such as the introduction of more "advanced" mathematical subjects - the so-called "New Mathematics." Nevertheless there is a lot of interest in these modern approaches to the teaching of mathematics. They are very much discussed, though less often tried. These remarks apply specifically to Jamaica, but I understand that the situation in the other West Indian islands is not very different.

University Teaching.

In 1961 the University of the West Indies (which serves all the former British West Indies) was still affiliated under a special relationship with the University of London, and the degrees it awarded were those of London University. Since then several West Indian territories have become independent, and the University of the West Indies is now, also, an independent University. The tendency has been however, to maintain the general pattern of the degree much as before, and only minor changes have been made in the mathematics syllabus.

There are two main streams of mathematics students; those reading for the "General" degree, in which mathematics is one of two or three subjects, and those reading for the Special degree in mathematics, who study mathematics alone. There has been a gradual increase in the emphasis on statistics in the mathematics course, and much more is included in this subject than formerly, both in the General and Special degrees. The development which is likely to make the biggest changes in the syllabus in the near future is the advent of computers in Jamaica. In 1961, there were none; now there are at least 8 in operation, and many more on order. This has opened up a new field of opportunity for mathematics graduates in programming, systems analysis, operations research, and so on. Accordingly courses in numerical methods and computer programming have been introduced into both undergraduate courses. In addition, optional courses in programming are offered, open to all undergraduates.

Conferences.

In 1963 a conference was held in Trinidad on the teaching of mathematics in the West Indies. This was attended by a large number of delegates, not only from the various West Indian islands but also from Canada, U.S.A., the United Kingdom and Venezuela. It featured many talks on teaching methods and techniques, film shows and discussions.

Conferences such as this, and smaller meetings such as those held at the University of the West Indies under the auspices of the Department of Education, have done much to stimulate interest in new and better approaches to mathematics teaching on the part of the more competent and progressive teachers. Unfortunately, until the general level of teacher training can be significantly raised (which will take a long time, at best) it is very difficult for these persons to make much headway either in applying or spreading the new methods.

MEXICO

Elementary Level

In Mexico, elementary education consists of six years and is constitutionally obligatory. The syllabi followed are the same for the entire country and are established by the Department of Public Education. This same Department formulates and distributes a series of uniform texts which, besides being free, are periodically revised and reconstructed. The uniformity of the text provides certain advantages, among which the following may be mentioned:

It assures unity of teaching, irrespective of heterogeneity of teaching personnel. It permits, in the general way, the immediate introduction of changes in order and content of the subject matter.

Elementary level teachers are trained in the normal schools most of which are state supported. Unfortunately the level of mathematics preparation achieved by the graduates of these schools is far from satisfactory and constitutes one of the greatest obstacles to the true structuring of mathematics instruction. In fact, at best, the training of these teachers does not go beyond the equivalent of the mathematics content of the first three years of the intermediate level; and what is worse, the teaching to which they are generally subjected is completely inadequate in content and orientation.

number of concomitant factors characterize this situation. On the one hand, there is a fatal separation between the normal schools and the other academic institutions of the country; and also, the career of the normal school teacher offers too little material reward to attract enough people with the appropriate dedication and ability. In other cases, the potential good development of the more able and more dedicated teachers is forfeited because of the extra work with which they must burden themselves in order to achieve a liveable economic status.

Nevertheless, a project has recently been inaugurated, still in experimental stages, which substantially modifies syllabi and teaching methods in mathematics at this level. This experiment follows the general lines of the modernization of mathematics teaching which have been put forth by various groups of educators and mathematicians in other countries. The first results shown by these experiments lead us to believe that it will shortly be possible to make these changes more universal.

Intermediate Level

The intermediate level in Mexico is divided between the so-called high school or prevocational school (three years) and the preparatory or vocational school (2 or 3 years). In contrast to what happens on the elementary level, the secondary level is not homogeneous, since the diversity of authorities and educational centers at this level result in a great variation in syllabi, textbooks, and teaching personnel.

Most secondary schools, that is, those which deal with the first three years of the intermediate level, also come under the Department of Public Education. Almost all their teachers are graduates of the higher normal schools, which suffer from the same problems, essentially, as the regular normal schools. Some of the teachers are professionals in various fields (engineering, chemistry, medicine, etc.) and teach only complementarily. The little time they are able to dedicate and the preparation they can manage does not permit the achievement of the desired results. The case of teachers who have not been able to finish their professional studies is also not uncommon.

The situation regarding study programs is similarly not very incouraging; they are outdated and hardly related to what preceeds and what follows.

Along the same lines mentioned in connection with the elementary level, pilot projects are being developed to try to correct this series of anomalies and particulary to try to incorporate some of the ideas of modern mathematics into the respective programs.

The situation is different in the two or three last years of the intermediate level (preparatory and vocational schools), since these are closely related to the universities, polytechnical schools and generally institutions at the professional level. The faculties here principally is made up of professionals. While some of these teachers consider their goals as educators as secondary to their professional work, those who do dedicate themselves entirely to teaching find themselves obliged to undertake too great a number of classes if they are to achieve a minimal economic status.

Nevertheless, it should be pointed out that positions are being established for full and part-time teachers which will permit the achievement of a satisfactory income without overloading of teaching hours as well as efforts to improve one's academic preparation.

Also university students of the last years of the mathematics major are being called upon to take charge of courses at this level; they are considered to be sufficiently trained to administer these programs effectively. In addition, there is a plan to extend official recognition to students who complete five semesters of the professional level of mathematics as competent to teach at the intermediate level.

For teachers at this level, fundamentally, courses are frequently given for bringing knowledge up to date. These courses are given by leading institutions of the country, such as the Mexican Mathematics Society, the Division of Sciences of the National University, and the High School of Physics and Mathematics of the National Polytechnic Institute. Here recognized specialists give basic courses in the various fields of mathematics.

Professional Level

With regard to the professional level, one must consider two situations: one relating to the teaching of mathematics in mathematics departments and the other relating to schools where mathematics is a propaedeutic subject (engineering, economic schools, etc.). In this respect it should be pointed out that in Mexico almost no institutions of higher education have independent mathematics departments: that is, the students of the professional schools take their mathematics courses in their own schools independently of mathematics departments.

At present Mexico has seven mathematics departments in various institutions around the country. In all of them, the mathematics major requires eight semesters of study; the programs are quite homogeneous and adapted to the needs of modern mathematics. The objective of these schools is to prepare researchers, teachers and specialists in the various applied and technological branches of mathematics. For the researchers, the quality of teaching assures a good foundation. For the applied specialist, heretofore somewhat forgotten, there is now a decided tendency to establish programs that are more flexible than the traditional ones and permit students to include various subjects of applied mathematics. As for teachers, it is understood that theoretically they should be provided for by the adequate higher instruction;

nevertheless, the relatively small number of graduating students is almost wholely absorbed by the universities themselves or research institutions and few indeed take up teaching in other institutions. This has had the enormous defect of restricting the influence of these well prepared mathematicians. We hope the situation will improve as the number of graduates begins to exceed internal needs, which will happen since mathematics schools are currently experiencing great growth in their student bodies.

The great majority of faculty members in the mathematics school are full time and involved exclusively in academic matters.

In order to increase the numbers of students in the mathematics schools and thus in the future eliminate the shortage of teachers and mathematicians in general, some measures have been taken. The number and size of scholarships, for example, have been increased; students of advanced standing have been named as professorial assistants; a national mathematics contest has been established for students who have finished the intermediate level. Also informational lectures on television and vocational orientation lectures in intermediate schools have been given.

Plans also are underway for improving the general level of recently organized mathematics schools and faculties as well as those of inadequate resources. For example, we have set up teacher exchanges, special courses and higher salaries.

With regard to mathematics instruction in engineering, chemistry, economics schools, etc., the situation is faulty and chaotic, it does not meet present needs of science and technology. Attempts have been made to totally restructure the mathematics programs of these schools but it is practically impossible to bring about new programs in view of the inadequate mathematical preparation of the respective faculties. As a curative, attempts are being made, wherever possible, to equip professionals through specific courses given by mathematicians. The definitive solution to this problem was pointed out earlier in the discussion of mathematics schools.

Graduate Level

Mexico currently has two academic centers which offer graduate level studies leading to master's and doctor's degrees and a third such center begins operation next year. The academic level of these centers is excellent, for the branches which they cover.

Candidates for any of the academic degrees given by these schools have at their disposal an adequate number of scholarships to permit them to devote themselves full time to their studies.

Publications

The Mexican Mathematics Society periodically publishes a Society Bulletin and a Mathematics Review. The Bulletin is devoted to the publication of mathematical research. It has two issues per year, forming one volume. The first series was published from 1943 to 1955; the second began in 1956. It is of excellent appearance and has a quite acceptable international circulation. The Review is an informational organ, principally of domestic circulation. It features articles of interest for teachers of various levels. The first issue appeared in 1957; two issues are published every year.

The National Polytechnic Institute publishes the Mexican Note (Acta Mexicana) of Science and Technology, which has as its goal the providing of a vehicle for research work as well as articles on revising subject content.

The Mathematics Institute of the university has been publishing the annals of the institute since 1961. They feature articles by researchers and visiting researchers as well as notes from seminars.

Meetings

The Mexican Mathematics Society organizes national congresses and regional assemblies approximately once a year. These meetings take place in different educational centers of the country in the hope of stimulating interest in mathematics among students and teachers as well as explaining the newest and most interesting aspects of mathematics.

In 1956, an International Symposium on Topology was held, in 1958, an International Symposium on Educational Differences, both organized by the Mexican Mathematics Society under the auspices of the National University of Mexico. The records of these meetings were subsequently published.

Needs and Suggestions for the Next Few Years

A number of needs follow from the previous paragraphs; but in any case, they may be summarized as follows:

Greatly encourage relations between professional mathematicians and teachers of mathematics; in particular the professionals should collaborate substantially in the preparation training and bringing up to date of the teachers; they should also formulate programs, designate textbooks, propose and execute translations and revisions of interesting books, etc.

Encourage the setting up of teacher training courses and modernization courses accessible to the greatest possible number of teachers and given by specialists.

Develop procedures for getting more and better students to enroll in the mathematics divisions and along with this find more and better opportunities for the graduates of these schools.

Particularly at the intermediate level it would be salutary to provide more full time and half time positions.

Wherever possible, encourage the establishment of autonomous methematics departments having charge of all the mathematics subjects of the respective institutions.

Finally, it would be well to strive for more recognition of the profession of mathematicians. Actually in our country the recognition has not been fully granted. For example, even the general Office of Professions of the Department of Public Education does not recognize it.

NICARAGUA

Elementary Education

The mathematics courses given at the elementary level provide the student with the following knowledge:

Arithmetic: numerical operations, fractions, decimals, three step problems, interest, discount.

Geometry: description of geometrical bodies, planes and solids.

The thoroughness with which these topics are covered depends on the quality of the school. It can nevertheless be assured that students graduating from the elementary level do have some general knowledge of the above mentioned topics.

High School Education

From 1959, the mathematics subjects of Nicaragua High schools were those shown in Table 1.

These courses were full year courses; the fifth year course was a general review of all mathematics courses.

In the academic year 1960-61 a reform was instituted in the content and organization of the mathematics courses; the reform was completed in the year 1963-64. The subjects are those shown in Table 2.

TABLE I

Content	First Year	Second Year	Third Year	Fourth Year	Fifth Year
Arithmetic	x				x
Algebra		x			x
Plane Geometry			x		x
Solid Geometry			x		x
Trigonometry				x	x

TABLE II

Content	First Year	Second Year	Third Year	Fourth Year	Fifth Year
Arithmetic	x	x			
Algebra		x	x	x	x
Plane Geometry	x	x	x		
Solid Geometry		x	x	x	
Trigonometry			x	x	
Statistics					x
Analytic Geometry					x
Differential Calculus					x

The first 3 years constitute the so-called basic studies; the last 2 years are specialized studies.

This last plan is currently in operation. In order to provide some idea of the thoroughness of the courses, the content of the fifth year mathematics is briefly described:

Algebra: Combinatorics, probability, review
Statistics: Measure of central tendencies
Analytic Geometry: The line and the circle
Calculus: Differentiation.

Programs

In the university the mathematics programs are undergoing a complete remodeling from the first to the last course. This is largely due to the assistance of a group of young mathematicians in the city of Managua, working on different university schools who have formed an unofficial working team which coordinates the content and sequence in the mathematics programs. The influence of these changes will be felt in the high schools as the teachers begin to teach topics of abstract algebra in the fifth year of high school.

Training In-Service Teachers

The training of intermediate level teachers is carried out through 2 courses given in the school of educational sciences which is part of the division of arts and sciences (humanities) of the National Autonomist University of Nicaragua.

These courses are:

I. A course called regular (ordinary) which lasts for four years and leads to the title of teacher of intermediate education with a speciality in physics and mathematics. The title of a higher level teacher (Licentiate) in educational science with a specialty in physics and mathematics can be obtained by the taking of more courses for a year and preparing a monograph.

II. A course called professionalization which is given in intensive form during the vacations and is the same as the regular. It was first given in March of 1963 to enable the teachers in the departments and those who worked in the evening institutes of the Capitol to attend. At present the length of these courses has been increased by a year.

Seminars are also given on different mathematics topics and are attended by students from the various university schools in the capitol and by university as well as intermediate level teachers. From October 1965 to April 1966, the following workshops were given:

Introduction to finite group theory, Introduction to symbolic logic, and Introduction to general topology. In March 1964, with assistance from CSUCA and NSF the seminar in modern mathematics was given in the division of physical and mathematical sciences by Professors Mariano Garcia (Puerto Rico), Bernardo Alfaro Sagot (Costa Rica), and Roberto Zelaya Blanco (Nicaragua).

TABLE III

MATHEMATICS COURSES TAUGHT IN THE MAJORS OF PHYSICS AND MATHEMATICS - DEPARTMENT OF SCIENCE EDUCATION

Titles	Teachers of Middle School First Year	Second Year	Third Year	Licenciate Fourth Year	Fifth Year
General Mathematics	x				
Analytic Geometry		x			
Calculus I		x			
Calculus II			x		
Differential Equations				x	
Vector Analysis				x	
Modern Mathematics					x
Statistical Inference					x
Mathematical Seminar					x

TABLE IV

DESCRIPTION OF MATHEMATICS COURSES IN THE SCHOOLS OF ENGINEERING, ECONOMICS, AND BUSINESS ADMINISTRATION

Titles	School	Semester					
		I	II	III	IV	V	VI
Higher Algebra	Engin.	x					
Analytic Geometry and Calculus	Engineering	x					
Calculus I	Engineering		x				
Calculus II	Engineering			x			
Differential Equations	Engineering				x		
Applied Mathematics	Engineering					x	
Statistics	Engineering						x
Mathematics I	Econ. + Adm.	x					
Mathematics II	Econ.+ Adm.		x				
Mathematics III	Economics			x			
Mathematics IV	Economics				x		
Mathematics V	Economics					x	
Statistics I	Econ.+ Adm.			x			
Statistics II	Econ.+ Adm.				x		
Sampling Theory	Administr.						x

Outstanding Publications

In the area of mathematics education, two main works have been published: 1) The Teaching of Mathematics in Nicaragua, Report presented by the Nicaraguaian delegation to the meeting of Central American University Teachers held in Guatemala February 1 - 5, 1965.

2) A proposed mathematics curriculum for the branch of the University of Nicaragua at Managua by Werner Rudolf Ketelhohn.

(A thesis submitted to the Graduate Faculty of North Carolina State University at Raleigh, January, 1965.)

In addition, two projects have been presented in the National University:

1. A Project for the creation of a central mathematics department for the university division of Managua, by Robert Zelaya, 1965.

2. A Project for mathematics department subjects for the division of the National University of Managua, by Werner Ketelhohn, 1966.

In the area of pure mathematics, various texts have been published:

1. Amplification of mathematics 1, Roberto Zelaya, 1963,
2. Amplification of mathematics 2, Roberto Zelaya, 1963,
3. Mathematical Information 1, Roberto Zelaya, 1965,
4. Amplification of mathematics (revised ed.), Roberto Zelaya, 1966

With the exception of publication III of all the above are at the fifth level of high school.

Principle Movements

1. The creation of a major in physics and mathematics in the school of educational sciences for training intermediate level teachers.

2. The creation of the Nicaragua association of university mathematics professors. This association was founded on the basis of the recommendations adopted at the Guatemalian meeting (February 1965) where the Central American Mathematics Association (ACAM) was created.

Our association seeks the departmentalization within the UNAN, and in a short time, as well, the creation of some vehicle to bring together intermediate level teachers; the establishment of workshops which would make available knowledge of mathematical currents and would thus further the reform of the study plans of the high school level.

Final mention must be made of the UNAN'S employment of full and part time faculty in order to service more effectively the various mathematics courses currently given.

4. With the aid of a loan from AID to the UNAN, a library of mathematics books has been created which will provide a reference service for students and teachers.

5. Summer courses in mathematics and physics are being planned to be given in March and April, 1967, in the school of Physics and the Mathematical Sciences. These courses will have a regional Central American character and will be given by senior professors of the universities.

Needs and Suggestions for the Coming Years

During the high rate of growth of the student body at the UNAN, the following may be said with reference to a beginning date of 1967:

1. That more and more mathematics teachers will be needed to handle the various courses, especially at the first year university level.

2. That it is necessary to have available an electronic computer to familiarize students with programming techniques and to aid in research.

3. That it is becoming clear that a Central American Mathematics Institute is needed, at the regional level, to train university level professors. This would avoid the waste of financial and human resources that would be implied if every country of the area decided to undertake this task independently. This task should be coordinated by CSUCA.

4. It is desirable to have a greater exchange of visiting professors in mathematics between the various Latin American countries, which could be sponsored by UNESCO with the center of Scientific Cooperation of UNESCO functioning as the coordinating

organism for Latin America.

5. To increase the number of trained mathematics teachers at both elementary and high school levels, seminars should be planned for the vacation months.

PANAMA

In the mathematical reform in Panama, particular mention should be made of two projects well along toward completion:

1) A new mathematics syllabus with corresponding analytic subjects for high schools (a project of the Ministry of Education), and

2) a new syllabus and related programs for the "Licentiate" (graduate degree in mathematics) and mathematics teaching degrees (project of the University of Panama.

Both projects were directed by Dr. Agustin Colamarca, Director of the School of Mathematics of the University of Panama. There exists also a plan for reforming content teaching on the elementary level.

University Education

Since the academic year 1965-66, the faculty of sciences of the University of Panama has been made up of six schools, among which is a School of Mathematics and a School of Physics.

The new program of the University School of Mathematics is as follows: 1) Preparatory Mathematics (Algebra and Trigonometry); 2) Introduction to the Fundamentals of Mathematics; 3) Modern Geometry; 4) Calculus and Analytic Geometry I; 5) General Physics; 6) Advanced Algebra; 7) Mathematical Logic; 8) Projective Geometry; 9) Calculus and Analytic Geometry II; 10) Mechanics, Electricity and Magnetism; 11) Differential Equations; 12) Modern Algebra; 13) Vector Analysis; 14) Advanced

Calculus; 15) Introduction to Statistics (one semester); 16) Numerical Calculus (one semester); 17) Introduction to Topology (one semester); 18) Introduction to Differential Geometry (one semester); 19) Mathematical Analysis; 20) Functions of Complex Variables; 21) History of Mathematics. In addition to English, two years of French, German or Italian are required.

As of November 1966, the above program is in force in the Freshman and Sophomore years, with a registration (for the two years) of 225 students. The students of junior and senior years are following the old Mathematics and Physics programs, with a total of 150 students (in the two years).

To aid in the implementation of the above program, the School has two visiting professors, Professor Ricardo Marino C. from Spain and Professor Jean Laplanche, sent by the French Government. A number of young professors from the university have attended special courses in Lima and Montevideo; visiting professors have given advanced courses for graduate assistants Such services were rendered, for example, by Professor Jose Reategui C., who remained in Panama from December 1964 to April 1965. An exchange arrangement exists whereby Panama university professors received scholarships from the French Government for doctoral level studies in France, while French professors replace them during their leave. Several graduate students are presently applying for post graduate fellowships at universities in the United States. We are envisioning also the possibility of extending the present Panama University curriculum through post-graduate studies in (pure) mathematics beginning 1970.

High School Education

The programs proposed for high school mathematics (1967) are geared to developing the concepts of sets, relation and functions. Various types of numbers, operations and properties are studied; various classes of relative numbers are derived from relationships among vectors. The concept of the "vector" including scalar and vector products and their properties is developed in the final years of the preparatory school program. In the last year of preparatory school, combinatorics, notions of probability, and basic concepts of differential and integral calculus are introduced. Every year includes study in geometry; the higher classes study notions of geometric transformations.

The following plan of action has been adopted for the most effective implementation of the foregoing program:

a) Preparation of texts:

The Ministry of Education has approved six specially prepared Arithmetic, Algebra and Geometry texts. They cover the entire first phase (3 years). The fourth-year Algebra and Geometry texts for the coming school year are in preparation.

b) Seminars:

The Ministry of Education and the School of Mathematics of the University of Panama have jointly organized and held a series of Seminars in Modern Mathematics, employing the services of various visiting professors:

1. Dr. Carlo Federici Casa of the National University of Colombia (Summer 1962)
2. Professor Jean Maumuse, currently French Scientific Attache in Madrid (Spain), (September 1964)
3. Dr. Jose Reategui C. of the National University of Engineering in Peru (December 1964 to April 1965)
4. Professor Jean Claude Laplanche, presently at the University of Panama and scheduled to be in charge of the Summer Seminar for 1967 which will deal with algebraic structures and geometric transformations.

Slow but satisfactory progress has been achieved at the secondary level, and significant improvement is expected in the coming years, as mathematics faculties acquire new teachers with more advanced and more modern training. At present, about 30 students graduate with degrees in Mathematics and Physics every year. This is enough for the current needs of the country.

Elementary Education

In order to equip our teachers to use modern mathematics texts (prepared in Guatemala by a group of Central-American and Panamanian teachers [ROCAP]) in our schools, the Ministry of

Education and the School of Mathematics from the University of Panama jointly organized a Summer Seminar in 1966. It was attended by more than 2,000 elementary school teachers.

The seminar took up set theory, using texts specially prepared by Professor Colamarco at the fourth year preparatory level. A new seminar for the Summer of 1967 is presently being prepared. It will study various systems of numbers, structures, and methodology.

For the school year, beginning April 1967, we expect to utilize the modern elementary texts of ROCAP through the fourth year of elementary school.

PARAGUAY

Mathematic education at High School level is governed by the following curriculum, fixed in 1958:

BASIC CURRICULUM

Arithmetic (1st course) 5 hours per week

Plane Geometry (2nd course) 5 hours per week

Solid Geometry (3rd course) 3 hours per week

Algebra I (2nd course) 3 hours per week

Algebra II (3rd course) 3 hours per week

SUPERIOR CURRICULUM

Trigonometry (4th course, Bachelor's degree, Business School) 3 hours per week.

Mathematics review (4th course, normal school) 3 hours per week.

As an application of trigonometry cosmography is given as a 5th course, 3 hours per week (for Bachelor's degree and normal school) and 2 hours (for business school).

This curriculum has not been modified since the year 1958.

The courses of the two first collegiate years of the institutions in which mathematics is taught (National University: Physical Sciences and Mathematics School, Science Institute, and Philosophy School; Catholic University, Philosophy School) are a review of the courses already taken in high school: Algebra, geometry and trigonometry. Beyond this, the study of Theory of

Equations linear algebra, analytic geometry, differential and integral calculus is initiated.

The college entering student body, as well as the students studying outside the country and the teachers agree that the performance reached in teaching and learning mathematics is at a low level due to many factors. As a reaction to this general feeling a new movement has emerged to increase the mathematical activity in the country, a movement reflected in the following facts.

A) NEW INSTITUTIONS

(1) The creation of the Institute of Sciences at the National University (1963), of which one of departments is Mathematics.

One purpose of the department is the training of the teachers in the knowledge of advanced mathematics, and advising and orientation in the teaching of mathematics in the secondary cycle. It is the first institution of the country that has introduced in its curriculum subjects such a s algebraic structures, metric spaces, concepts of general topology, functions of a complex variable.

Among the various activities developed by a newly appointed mathematician it is worth-while to mention the following courses: Boolen Algebra, linear programming, approximation of functions, linear algebra, and functions of a complex variable.

(2) The creation of the mathematics career at the Philosophy School, of the Catholic University (1964). Its plan of study consists of 4 years of mathematics and in addition a Bachelor's Degree in the training of teachers of secondary mathematics.

In 1966 The Normal Teachers School No. 2 organized, with the help of the Ministry of Education and the Sciences Institute, a course on Set Theory for professors of secondary education. Its length was 16 hours (2 hours per week), was attended by 23 professors, and was financed by the participants themselves.

The "Academia de Fisica del Colegio Nacional de la Capital" also organized in 1965 and 1966 short courses of two months duration (3 hours per week) to train students of the senior year interested in entering some of the technical schools or studying overseas. Notions of analytic geometry and differential and integral calculus were given. It was financed by the assistants.

Other brief courses were developed by the Sociedad Matematica Paraguaya, for members and university students.

C) LECTURES

Since its founding in 1964, 27 conferences have been given at the Sociedad Matematica Paraguaya, some of which are worth-while mentioning.

I. A Mathematical Week. Consisting of a cycle of 7 addresses o S. Sispanov: His life and works. Teaching of mathematics. The principles of the development of mathematics, Finite Geometry, Metric Spaces. The axiomatic method in Physics. The use of computers on Engineering.

II. A Mathematical Week. Consisting of a cycle of 10 lectures Julio Rey Pastor on Coordinability. Boolen Algebra. Syllogisms. Linear Programming. Calculus of variations. Laplace transforms. The use of Graphical analysis in Electricity problems. Hyperbolic Paraboloid. Scientific conception of the Universe. Nuclear Explosions effects.

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III. A Cycle of 4 Lectures on computers, its organization, foundation and applications by Garcia Camarero, and the use of computers in verbal protocol analysis (L.I. Ramallo). Also at the Sciences Institute many lectures have been given, among the most important: The didactics of Mathematics (E. Ranucci), Automation and Control, Present Panorama of Computers (J.C. Valle), A chapter of applied mathematics: logic (O. Dodera), Mathematics structures (O. Dodera).

IV. A Cycle of 6 Lectures about aspects of modern mathematics specially directed for high school teachers. This cycle was jointly organized with the teachers of the Normal School No. 2.

D) PUBLICATIONS

The most important tracts published in Paraguay are:

Notions about Set Theory and real numbers. J.L. Benza 1964.

Congruences and Residual Clases. M.C. Gomez Vantre 1964.

Boolen Algebra. E. Garcia Camarero. 1965.

Set Theory. H. Feliciangeli and M.C. Gomez Vantre 1966.

Communications. Three publications of the Magazine of the Paraguayan Mathematic Society (appearing without determined period).

Geometry II. L.A. Santalo 1966.

Exercises and Analysis Problems II. B. Hochsztajn and A. Lopez 1966.

Observation

Due to lack of finances the publication of a mathematical magazine for teachers of high school education is not possible as yet.

All the factors indicate a great interest in provoking an improvement in the teaching of mathematics and that the proper moment to accomplish it has been reached. Nevertheless the transformation has not yet come about.

In order that the effort for improvement be effective, attention must be given to the three following points and to the economical problems involved:

1) The training, orientation and qualification of the teachers. For this purpose it is of importance to give brief courses for teachers in service not in an isolated and sporadic way, but periodically, according to an elaborated plan with the help of experts, for no less than three years.

A program of training of teachers and college professors overseas on a scholarship basis would be an unquestionable contribution. It is also of great importance to have an improvement of teachers remuneration.

2) The Creation of a mathematics library. At present there is no one worth-while library in the country. The total different kinds of mathematics books from the main private and public libraries of the country hardly reaches 700 volumes. The number of journals regularly received is minimal.

3) Restructuration and actualization of effective curricula at the high school as well as at the University.

PERU

The activities undertaken in Peru for the improvement mathematics instruction during recent years have been carried essentially by a small number of university professors who interested in this problem since 1961 and who were inspired primarily by experience of the U.S. and the declarations of the First Inter-American Conference on Mathematics Education. The fields in which these activities have been carried on are the following:

1. Improvement of High School Teachers

This activity has been going on uninterrupted at least every year since 1961. It has consisted of course of an average duration weeks, planned with the aim of providing a group of in-service teachers with concrete information about certain basic knowledge of analysis, algebra, and geometry. The teachers were brought together and received rigorous theoretical and practical instruction. They were generally divided into two groups according to their background; those who finished the first level one year went on to the second the following year. During four consecutive years the courses had an international character, being assisted by the Peruvian Ministry of Public Education, the National Science Foundation of the U.S., the Pan American Union, the Ford Foundation, the Peruvian Institute for Educational Development, the School Mathematics Study Group and the National Engineering University. In the time that has transpired more than 500 teachers have received instruction, including information about the work done in other countries to modernize mathematics instruction. In particular, they were learning c ontent through the textbooks published by the SMSG which were the object of discussion at seminars. As a consequence, these publications are now known to most Peruvian teachers.

It is interesting to know that the establishment of the summer institutes, organized within the improvement program, awakened interest in the establishment of similar courses by other sources, for example the summer course of 1965 given at the National High University of San Marcos. It also has given rise to efforts to extend this activity to other fields. Thus, in the following years the Ministry of Education and centers of advanced education have given courses for high school teachers of physics, chemistry, biology, etc.

2. Institute for the Promotion of Mathematics Education (Instruction) (IPEM)

In order to assure continued efforts toward the improvement of mathematics instruction, a private non-profit institute was established. Its activity was directed principally to the organization of Summer Institutes such as we have described, to editing texts for high school teachers, and to establishing an essential library for them.

The IPEM is currently working diligently to organize mathematics and physics clubs in all of the important cities of the country. These are private non-profit clubs and their activity centers around organizing new institute s for the improvement of high school teachers, libraries, and laboratories as needed to further mathematics and physics, in collaboration with the universities. In Cuzco and Arequipa these clubs are already functioning.

3. Mathematics Texts for High School Teachers

A certain number of books for training high school teachers have been published. These texts are used in the summer institutes and have been widely circulated throughout the country and in some foreign countries. They deal with fundamentals of algebra, [illegible] geometry, and analysis. In view of the enthusiastic reception enjoyed by these publications it is proposed to extend them to other fields.

4. The Institute of Pure and Applied Mathematics of the National Engineering University (IMUNI)

The establishment of this institute in 1962 constituted one of the most progressive steps taken in the country for furthering advanced mathematics studies. It is at present linked with the School of Mathematics and Sciences of National Engineering University but it is operating independently of the teaching activities, with the objective of furthering research in pure and applied mathematics. Notwithstanding this fact, all its members are professors of the School and give part of their energies to it. In the short time that it has been active it has made considerable progress. It has its own building which was constructed with support from the Ministry of Education and its specialized library with more than 5,000 volumes established by a substantial grant from the Ford Foundation and gifts from the Embassies of France, Germany, and Japan. It is undoubtably the most complete library of its kind in Peru. In view of the enormous demand for higher education and mathematics, the university has recently decided to enlarge the plant of the institute.

5. The Regional School of Mathematics (EREMAT)

Recognizing that all efforts of improvement in mathematics teaching in Peru are significantly limited by the absence of qualified personnel to direct it, the IMUNI undertook to establish a studycenter to provide higher level intense instruction to carefully selected groups of mathematics teachers with university degrees in order to equip them to do their instruction more effectively. This resulted in the establishment of the Regional School of Mathematics founded this year. The 20 Peruvian and 10 foreigners from Chile, Argentina, Columbia, Costa Rica, who are participating have stayed at the School the entire year working full time on two levels. The project has been made possible by support from the NSF, the Ford Foundation, the Peruvian Institute of Educational Development, and the National Engineering University. Results have clearly been excellent, not the least important one being the contribution to better education of mathematics teachers for various Peruvian Universities and higher normal schools. In the coming year this work will be continued; it iw viewed as the most important work which has been undertaken to date. In this manner we intend to contribute to a solution of the problem of educating mathematicians to meet the great demand for university professors and teachers for normal school, technical schools and state institutions of education.

6. Other Activities in the Universities and Normal Schools

Among the institutions of the higher level who are collaborating in the development of mathematics instruction, the following merit mention:

The University of Trujillo, where the Mathematics Department, just recently created, gives direction to mathematics instruction of the entire university and especially in the School of Education.

The University of Huamanga, which has been training high school teachers for the last few years, giving them a sound background and concerning themselves with improvement of its own faculty. Two of its professors are presently studying in the regional Mathematics School.

The University of Cusco which, in collaboration with the National Engineering University, has completely revised its study plans for the mathematics section and in the Summer of 1966 began a program of improvement for a group of its teachers, one of whom is studying in the regional Mathematics School.

The Catholic and the Agrarian Universities which are organizing mathematics departments. The former schools also have a school of education which is expanding its activities in the area of teacher training in the sciences and particularly in high school level mathematics

PUERTO RICO

In Puerto Rico, we are aware that the marked development of science and mathematics during the present century have made it necessary for our people to have a better and higher education in these subjects. We have, therefore, organized a series of programs related to the training of mathematics and science teachers as well as the study plans for all levels of instruction. The following resume sets forth the most significant achievement in the field of mathematics during the period 1961-1966:

Curriculum Changes

For the secondary school level, in 1961, we began using English textbooks for mathematics developed by the School Mathematics Study group for academic mathematics (SMSG). During the years 1962-1963, the Department of Instruction in Puerto Rico started an experimental program using Spanish translations of the SMSG texts for the 7th and 8th grades as well as the upper and lower first algebra courses for the 9th grade and the geometry for the 10th grade. The use of these materials has been constantly extended to the various school districts of the island and according to recent statistics of the Department of Instruction, at present there are around 14,000 students participating in the program.

In the public schools of Puerto Rico from the 11th grade on, mathematics is an elective subject, and as a result, many students graduate having had only the first course in algebra and a course in geometry. Nevertheless, some now take courses in algebra and trigonometry in the 11th grade. The Department of Public Instruction plans tomake available elective courses in mathematical analysis, mathematics for computers, probability, and statistics, and eventually linear algebra and matrices for the 12th grade. In a number of private high schools, the syllabus already includes a course in Cartesian geometry and some introduction to infinitesimal calculus.

In the University of Puerto Rico, since August 1961, the school of pedagogy requires six credits in mathematics of all students registered in its study program. The special course formulated for this requirement was developed by the Department of Mathematics in collaboration with the School of Pedagogy and aims basically to present the fundamental ideas on methods of mathematics. The subjects taken up include sets, real number system, equations and inequations, numeration systems, modern geometry, and an introduction to Cartesian geometry. The logical structure of mathematics is underlined in the course and calculating techniques are deduced from the property and structure of the real number system. Previously education students were required to have a course in the methodology of instruction in mathematics and sciences.

At the university level also, various institutions have revised their mathematics course in the first year. A course combining algebra and trigonometry tends to be substituted for separate courses in university algebra and trigonometry. In the present program the concept of function is emphasized and

Trigonometry is studied from the analytical rather than the numerical point of view. Other curriculum changes at the university level include the requiring of a course in abstract algebra for students taking their undergraduate degree in mathematics, the intensification of statistics and mathematical analysis courses and the offering of introductory courses in mathematical logic and topology in the undergraduate program.

Retraining of Teachers

Since the year 1957 and under the auspices of the National Science Foundation, about 40 institutes of mathematics and sciences have been set up for teachers of elementary and secondary level. The Department of Instruction of Puerto Rico has given its cooperation and support to this activity. The institutes have been of 3 basic types: Summer Institutes, Academic Year Institutes, and Institutes for in-service teachers. More than 1200 teachers have participated in these programs. In the summer institutes for in-service teachers training is generally offered in modern courses, and biology, physics, mathematics, and chemistry are developed by the various experimental curriculum study groups in the United States. The Academic Year Institutes provide intensive training to groups of teachers carefully selected who may eventually be directors in the modernization of syllabi of sciences and mathematics in the entire island of Puerto Rico. At present, two of our mathematics supervisors are graduates of academic year programs.

Three years ago, the Department of Instruction received a grant from the Ford Foundation for the establishment of a system of curriculum centers in the various regions of the island. Three centers were organized, one in each of the principal three cities. These centers already have made important contribution in the following areas:

a) The preparation of teachers through seminars, lectures, professional meetings and special courses in mathematics and science. In this aspect of the centers, we enjoy the collaboration of consultants from the various universities of Puerto Rico.

b) The preparation of curriculum materials for enrichment in mathematics and science courses. Here we have the help of the teachers within the centers, coming from elementary and high schools. Most of them have received some training in the various mathematics and science institutes. Once the materials are prepared, they are reviewed by consultants at the university level and by the directors of mathematics programs of the Department of Instruction of Puerto Rico.

The projects described above have substantially contributed to the betterment of the education and retraining of mathematics and science teachers in Puerto Rico, but the situation still requires considerable attention.

Significant Publications

Some teachers from the University of Puerto Rico, among them Professor Tomas Rodriguez Bachiller, Eugene A. Francis, Augusto Bobonis, Carlos Abreu, Francisco Garriga and Mariano Garcia have contributed to the development of Spanish language materials in modern mathematics by publising books, translations, and revising existing books.

In 1965, the mathematics dictionary English-Spanish, Spanish-English, was published. Before that a translation was done in Puerto Rico of a work called Integrated Course of Algebra and Trigonometry by Fisher and Ziebur and revisions were made in the translation of Modern Algebra and Trigonometry by Vance.

During the last five years we have also translated the following texts:

a) The SMSG First Course in Algebra

b) The SMSG Geometry

c) Modern Geometry by Moise and Downes

d) Mathematics Three and Mathematics Six of the Laidlaw Mathematics Series (eventually Mathematics Four and and Mathematics Five will also be translated.)

e) Mathematics One, Mathematics Two and Mathematics Six of the Silver Burdett Mathematics Series (eventually Mathematics Three, Mathematics Four, and Mathematics Five will also be translated.)

The first five of these books are being used in elementary and secondary schools in Puerto Rico and the translation of the Fisher and Ziebur book is being used in the first college year in Mayaguez, Rio Piedras and Humacao branches of the University of Puerto Rico. These translations are of great assistance in bettering mathematics instruction in our country.

In addition, the curriculum center of the Department of Instruction has prepared notebooks from grades 1 to 6, information bulletins, and work unites with which to supplement the textbooks.

Exceptional Programs

Since 1961, the University of Puerto Rico has had special mathematics programs sponsored by the National Science Foundation for outstanding high school students. In this program a special niversity level course in algebra and trigonometry is offered o give the students the opportunity of beginning advanced athematics studies while they are finishing high school. At resent various high school teachers are participating in the rogram to observe at close range the actions of the participating tudents who face new concepts and also to familiarize themselves with he new areas of focus in mathematics teaching at the university evel. In addition, several summer programs have been offered for utstanding students. Some of the subjects included in these rograms are computers, analytic geometry, matrices, elementary nctions, and statistics and probability.

Under the sponsorhip of the National Science Foundation a program of lectures is being conducted on mathematics and scientific subjects for teachers and students of high school level. These lectures are given by university professors and seek to broaden the knowledge of teachers in the subjects they teach and to interest students in the field of mathematics and sciences. The University of Puerto Rico and the Department of Public Instruction jointly ran a program whereby selected students from the public schools of Puerto Rico attended some of the regular courses of the first year at the university and received university credit if they satisfactorily passed the courses under the same conditions as those applying to university students. This program includes mathematics though it is not limited to it.

Another program that has been successful has offered to students with unusual mathematics ability an introductory course in mathematics given the summer preceeding their admission to the university. Thuse these students can register in more advanced mathematics courses for their first university year and have the opportunity of getting more mathematics education during their years in the university.

As an integral part of the institute programs we have been fortunate to enjoy visits from distinguished mathematicians from the United States and abroad who have given lectures and led discussions with faculty members in mathematics. All levels have benefited from these contacts.

During the last two years, the Department of Instruction has experimented with the use of mathematics courses on television for 7th and 8th grades which do not use the SMSG texts. The results of this project have been very satisfactory and the program will probably be repeated during the coming school year. Some experimentation is also going on with programmed instruction at the elementary level. At present, units are available on set theory and the addition and subtraction of fractions; other units are in preparation.

Recommendations and Suggestions for the Future

Recently a Commission on Mathematics was created under the auspices of the College Entrance Examination Board's office of Puerto Rico to carry out researches and studies on the instruction in mathematics in Puerto Rico at the high school as well as the university levels. These studies will be done during the next two years with aim of instituting in Puerto Rico a program of experiments of academic improvement in mathematics. It is hoped that the work of the Commission will have positive results setting up models to follow for developing syllabi at the various levels and for improving the quality of mathematics instruction in Puerto Rico.

REPUBLIC OF EL SALVADOR

The Current of Mathematics Thought

At the beginning of 1962, modern mathematics first became known in our country; the phenomenen has since been called "the revolution in mathematics." Before that date there certainly had been ideas about the new thinking in mathematics, but it never amounted to more than isolated instances of instruction of a few topics.

Since then institutions like the Ministry of Education, the School of Engineering and Architecture, and the School of Economic Sciences have become aware of the new direction that this important branch of science should take. This awareness has been assisted by the results of the first Inter-American Conferenc e on Mathematics Education in America, held in Bogota, December 1961. It must also be pointed out that publications like the textbooks of the SMSG and the so-called Revolution in Mathematics also contributed significantly to this awakening. Contributions also came from the meeting at San Jose, Costa Rica, in 1962, for Advancement in Science; the first Conference of Central American Mathematicians held in Guatemala in February 1965 as well as the summer courses which were held in San Jose, Costa Rica, and Lima, Peru. Finally, visits of some delegates from El Salvador to the United States and various other countries of South America have had a very salutary effect.

We can briefly summarize the comprehension of modern mathematics with respect ot its unifying characteristics for the entire discipline. We agree that logic is the idea system for building and that set theory is the building material; then we seek the goal of mathematics through specific mathematical structures. It may also be stated that in the methodological system of instruction it is indispensible that the axiomatic method be employed.

As far as content is concerned, certain things which were important in the past must be dispensed with, things that were purely encyclopaedic and had to be sacrificed to make place for more useful knowledge in the modern world in which we live; Topics such as projective geometry, linear programming, etc. Our directive personnel and a small group of teachers have been particularly concerned with these matters.

Problems of Reform

Nevertheless, the implementation of the current of modern mathematics has been difficult if not impossible, principally because of the inadequate training of a sufficient number of teachers. Intermediate level education in our country is imported by teachers with traditional training and by graduates of the Advanced Normal School. These teachers are not as numerous as the country needs which makes it necessary to use preparatory school graduates with little university training. Both the university and the Ministry of Education do send small groups of persons abroad to study and indeed, our needs would be greatly served if we could effectively establish a good scholarship plan which would help produce competent personnel for the teaching of mathematics. Both the university and the government of El Salvador are lacking the funds necessary to send adequate numbers of teachers abroad to study. In addition, it is impossible to contract capable foreign personnel since experience has shown that it is very diffuclt to obtain the services of academic professors at a high level. The university of El Salvador has tried many times to serve the Ministry of Education by establishing training courses for the teaching personnel but unfortunately to date these efforts have been unsuccessful. On some occasions lectures and orientation series have been given and have spread some restlessness and a desire among teachers to become more involved in the new methods and knowledge in mathematics. Also our university has satisfactorily given extension courses that have been successful. It is now clear that those in charge of intermediate level education must have more knowledge to teach than that which they do have. They should have at least the equivalent of a masters degree in mathematics. It is also important to mention that teachers from various schools and national institutions have enrolled as regular students in the university and others have become instructors or assistants in the subject. This in spite of the lack of understanding of the principals or directors of these schools and institutions for such extended study.

Instruction at the Lower University Level

The first two years of university studies are characterized as pre-university preparation and are preferred to as common area studies. After these studies, depending on the motivation of the students, their demonstrated merit, and on the screening process that is done at the end of the second year, students undertake the specialized professional studies of their majors.

These two years are divided into four semesters, each lasting about ninety hours. Of these, 60 are devoted to theoretical exposition and 30 to laboratory and mathematical exercises. The aims of university education can be classified as having four functions: Theoretical exposition, mathematical exercises, library work, and research. Following is a resume of our programs:

First Semester:

Mathematical logic, theory of sets, theory of numbers, (primes, divisibility, sums, facotring, mathematical induction, Pascal's triangle, sequences, binomial theorem), extension to integers, to rationals, to irrationals; the set of real numbers and the numerical real line, complex numbers (with applications to trigonometry, vectors, and the Gaussian rotational field, theorem of DeMoivre), vectorial algebra, applications of vectors to geometry and physics.

Second Semester:

The study of functions, polynomials, inverse functions, direct and inverse trigonometric functions, exponential functions, logarithmic functions, and linear programming; Permutations and combinations; Theory of groups, rings, and fields, matrice calculations, theory of determinants, applications of inverse matrices.

Third Semester:

Applications of vectors to the geometry of space, the straight line, the conics, problems of tangents and diameters, study of limits, analysis of continuity, theorems on limits of all classes of functions, continuity of function, the derivative, applications of the derivative, (critical points, maxima and minima, curvature, differentials), mechanics of differentiation.

Fourth Semester:

Theorems of existence, theorems of Rolle and mean value, applications of mean value theorem, L'Hospital's rule and indeterminate forms, introduction to integration by definition of area, Riemann Integral,short table of integrals (du, x^m, $\frac{du}{u}$, sin u, cos u, e^u), uses of the table of integrals, special techniques of integration (algebraic substitution, trigonometric substitution, integration by parts, the art of integration) table of integrals, applications to geometry and physics, series, expansion of functions, Taylor's and MacLaurin's formulas, applications to expansion in series, interval of convergence, partial differentiation, multiple integration, ordinary differential equations.

Beyond the programs that have been indicated for the two years of professional major there are studies in statistics based in set and probability theory. For the engineering and chemical science majors there is also an extension of specific mathematical applications in a modern course in operational calculus which we call mathematics fiv e. In the industrial engineering major, numerical analysis, and linear programming are taught based on Dantzig's method.

Conclusions

We feel able to say that mathematics education has progressed considerably in El Salvador though we expect to obtain our best results within another five years. Our university is seriously concerned with establishing an effective scholarship plan for studies abroad as well as brining the assistance of competent foreign personnel to give the necessary mathematics courses for our teachers to acquire more academic credit and appropriate competence which our country needs. Similarly it must be mentioned that the same efforts are being made by the Ministry of Education, although we think it more significant that the Ministry be the agency that points out the problems and tell the institutions which can lend valuable assistance what they need to have in their developing programs. We feel obliged to express our gratitude for the help given by AID in the area of mathematics books which have greatly strengthened our university library. We hope to continue to receive bibliographical material and other publications which will keep us abreast of the advances being made so swiftly in mathematical knowledge.

TRINIDAD AND TOBAGO

Introduction

Orientation to new programmes in Mathematics started in Trinidad in August, 1963. Before this date only traditional programmes were pursued.

In August, 1963, the Mathematical Association of Trinidad and Tobago in cooperation with the University of the West Indies sponsored a conference for secondary school teachers and lecturers at Teachers' Colleges to discuss certain new trends in Mathematical education.

This conference stimulated a series of courses. The first set of courses was conducted from September, 1964, to July, 1965, when an American Professor visited the University of the West Indies. It was felt by many teachers who attended these courses that since in Trinidad secondary school candidates were prepared for examinations of Cambridge University and the University of London, it was desirable for them to know of mathematical education trends in the United Kingdom. Thus another series of courses was conducted during the summer of 1965 when an English Master from the School Mathematics Project of Southampton, United Kingdom, visited the University of the West Indies. As a result of these orientation courses the need was felt for material which the Trinidad teachers could easily understand. This prompted certain writings which were published for our teachers. The need was also felt for changes in curricula. This need was satisfied in certain institutions.

Movements and Conferences

The Mathematical Association of Trinidad and Tobago in cooperation with the University of the West Indies sponsored the first conference in Trinidad to discuss recent trends in Mathematical Education. The conference was held in the summer of 1963, and was attended by representatives from:

Barbados.....5	Guyana......3
Grenada......2	Jamaica.....1
Nevis........1	St.Kitts....1
St.Vincent...1	Surinam.....2
Trinidad.....32	Venezuela...2

Most of the representatives were secondary school teachers. Visiting professors for the conference were from Canada, United Kingdom, United States of America and the West Indies.

The organization of courses and the dissemination of knowledge of the new programmes were then undertaken by the Institute of Education of the University of the West Indies.

As the Institute of Education is directly concerned with programmes at schools and teachers' colleges it was appropriate for this body to further the work in mathematical education in the territories served by the University of the West Indies.

Retraining of Teachers and Professors

During the conference on Mathematical Education, held in the summer of 1963, three professors were invited to join with professors of the University of the West Indies to give lectures on recent trends in mathematics. They were:

Professor Ralph James - University of British Columbia, Canada

" Robert Wisner- University of New Mexico, U.S.A.

" Bryan Thwaites-University of Southampton, U.K.

Many of the other participants were secondary school teachers and lecturers at teachers colleges. This was the first attempt in Trinidad to retrain some of our teachers.

With the visit of an American Professor to the University of the West Indies in September, 1964, on a Fullbright Lectureship, a series of courses were arranged.

The first course was conducted from the 26th September, 1964, to 31st October, 1964, when fifty-three candidates took part. The second course was conducted from the 2nd November, 1964, to 12th December, 1964, which another sixteen candidates attended.

In these courses the following topics were presented: Number Systems and their properties, Sets and Geometry. Following these courses a seminar was held from 4th to 8th January, 1965. Here teachers were requested to discuss, and were expected to write, topics which they felt would be suitable for their students. There were three groups of teachers and each group planned for one of the divisions:

(a) the lower secondary school (age group 12 to 14 years);

(b) the upper secondary school (age group 14 to 16 years);

(c) teacher college students.

Twenty-seven teachers attended the seminar and represented seventeen institutions.

In all we had an attendance of sixty-nine teachers in orientation courses and twenty-seven teachers in the seminar. Twenty-eight institutions were represented out of a total of forty-five institutions in the territory of Trinidad at that time.

At the end of these sessions teachers were requested to express their views on introducing the new trends in mathematical education in their schools. The expressions were:

(a) they needed further courses;

(b) since the secondary school candidates write the examinations of the University of Cambridge and the University of London, they should get some ideas of current trends in mathematical education of the United Kingdom.

Their first request was met when during the Summer of 1965 a Mathematics Master associated with the "School Mathematics Project" of Southampton visited the Institute of Education of the University of the West Indies. Their second request was partially met by the Extra Mural Department of the University of the West Indies when it made arrangements with the Mathematics Department to sponsor evening courses. These courses were at two centers for geographical reasons and were conducted from October, 1965 to June, 1966.

During the summer of 1965 two series of courses were conducted. One mainly for the secondary school teachers who requested knowledge of the recent trends of mathematics in the United Kingdom. At these courses emphasis was given to Transformation Geometry.

Another series of courses were conducted for teachers of children within the age group twelve to fifteen years who would not enter secondary schools, but would have three years' further education after primary school. These courses were attended by teachers from about twenty percent of these schools. Topics discussed at these courses were: Sets, Numbers, Symmetry, and Operations. Psychological principles were also discussed.

Another series of courses with respect to retraining of teachers was given from September 1965 to May 1966, at teachers colleges in Trinidad. These courses were given by a member from the Institute of Education and were conducted with the aim of helping the lecturers and teachers to introduce some of the new trends in mathematics in their curricula.

Curricula Changes

Difficulties which affect a change of curricula with respect to Mathematics are:

(i) obtaining staff;

(ii) getting staff to remain for a sufficiently long time to carry through a five-year course at the secondary level in Trinidad. At the secondary school level despite the several courses given for teachers, it is difficult to say whether any significant change has occurred. Out of a total of about forty-two schools, about six schools are teaching a "new mathematics" curriculum.

Beginning in 1964 the University of the West Indies' programme

Mathematics for the Bachelor's degree required a reasonable amount of the "new Mathematics" including Elementary Mathematical Logic, Elements of Set Theory, Boolean Algebra, Algebraic structures, Linear Algebra and Vector spaces.

It is hoped that with the increase of graduates from the University more changes will take place.

At the Teachers' College level, where the course is for two years, and where much assistance was obtained from Canadian lecturers through the Canadian Technical Aid programme, significant changes were made.

The Institute of Education in consultation with the Teachers colleges prepared a new programme in mathematics which was approved by the Ministry of Education. This programme thought started before was made official in February 1965 but effective from September 1965. The year September 1965 to July 1966 was considered as a transitional year.

Publications

To assist to effect changes certain publications were made. Many teachers expressed the view that the "New Mathematics" programmes were too abstract, and that relationships to concrete situations were desirable.

In an attempt to help teachers the following publications were written:

1. PRINCIPLES OF THE TEACHING OF MATHEMATICS by Sair Ali Shah, published by Longmans, Green and Co. Ltd., 48, Grosvenor Street,

London W. 1.

2. IDEAS IN MATHEMATICS by Sair Ali Shah and published for the Ministry of Education of Trinidad and Tobago.

This series is distributed to all primary and secondary schools and teachers colleges. So far the topics treated are: Numbers; Operations; Measurement - concepts, units, Instruments; Variation which includes Number Pairs, inequalities, linear programming. Others in the series to be published will include Sets, Matrices, Algebraic Structure; Geometry - Euclidean transformations, non-Euclidean ideas, topology; Statistics; Trigonometry.

Present Status of the Mathematical Curricula at Secondary and Lower University Levels.

At present there are about six secondary schools where a new mathematical curriculum is used.

The children within the age group, twelve to fifteen years, and who were not selected to enter secondary schools, but who are given three years of education beyond their primary school course are introduced to many new ideas. New Mathematics curricula are now official at the Teachers' Colleges in Trinidad.

Future Needs and Suggestions for Action

Courses

1. It is proposed to give short courses to primary and secondary school teachers.

2. Because of the large exodus to industry of people who are qualified to teach mathematics, it is felt that an increased number of

persons should be encouraged to read mathematics courses. To meet the ever increasing demand for teachers of mathematics it may be desirable to have a one-year full time course for teachers who completed their teacher training courses at the Teachers' Colleges. This course may be sponsored by the Institute of Education of the U.W.I., St. Augustine.

<u>Research</u>

There are a number of fundamental questions we may ask with respect to a programme in mathematics. Research programmes to answer these questions are desirable. We have already started some of this research and it is hoped that with our limited resources further attempts will be made to seek answer to these questions.

THE UNITED STATES OF AMERICA

1. With the report of the Commission on Mathematics of the College Entrance Examination Board in 1959, and the writings of the School Mathematics Study Group, consisting of textbooks for school years 7 through 12 which appeared in 1959-63, the program in secondary school mathematics has become fairly stabilized in the U.S.A. The program can be described as follows:

2. The study in Elementary schools will complete the arithmetic of whole numbers and fractional numbers, including systems of numeration with stress on decimal notation, computational skills in the four fundamental operations at adult level performance, factors, prime and composite numbers, positive integral exponents and applications to per cent and common quantitative problems. In geometry, the study includes the physical understanding and recognition of common figures (plane and solid), the nature of measure and its application to length, area, and simple volumes, and the intuitive ideas of congruence, symmetry, parallelism and perpendicularity. Throughout the entire elementary program, the intuitive ideas of sets are used to clarify the basic concepts.

3. School Year 7. Study of an informal structure of the whole numbers; systems of numeration; rationalized computational algorisms; factoring and divisibility; the study of fractional numbers; informal Euclidean geometry including constructions; applications to percent, distance, area, and volume; and introduction to algebra.

School Year 8. Study of an informal structure of the rational numbers, the Pythagorean theorem and square roots; irrational numbers, the real number line; the negative numbers; introduction to the solution of equations and inequations; finite number systems, statistics and graphs, indirect measurement and numerical trigonometry. Introduction to deduction and proof, informal Euclidean geometry.

School Year 9. A year of study of elementary algebra from a more formal study of number systems, the use of sets and set operations, the usual operations and transformations on polynomials and rational expressions, the solutions of equations and inequations; the extension of deduction and proof.

School Year 10. A year of strong deductive axiomatic geometry, plane and solid, using the properties of real numbers. The material covers the usual Euclidean synthetic geometry and introduces rectangular co-ordinates. Real numbers and their properties are used in geometry to make more precise the ideas of betweenness.

School Year 11. A year of extended study in Algebra including a formal study of the system of real numbers; functions and mappings; the functions called linear, quadratic, rational, exponential, logarithmic, angle, and circular. The nature of an inverse function. The related algebraic solution of equations and inequations, and transformations of these functions.

School Year 12. Since most schools hesitate to introduce a course in calculus, this year of study is still in a state of flux. Most commonly the study is a continuation of the study of some of the following topics: algebraic functions, limits and continuity, combinatorial analysis, probability and statistical inference, analytic geometry, and matrix algebra, including simple vector spaces. The study of trigonometry and solid geometry is extended.

For the more able students the above program is completed by the end of School Year 11 and the twelfth year is given over to a study of analysis including differential and integral calculus and analytic geometry.

4. There are major movements going on in the reform of the elementary school program that now promise better prepared entrants to the secondary school. The arithmetic is developed as an intuitive structure of the properties of numbers, the study of space is started in the first school year. After a few years it may be necessary to revise the secondary school program so as to include many of the proposals of the Synopses on a Modern School Program. For example, vectors receive little or no attention in the present secondary school program in the U.S.A.

5. A large number of monographs have also appeared written specifically for teachers and for talented students with an interest in mathematics.

6. There is no differentiation in the study of mathematics by non-scientific college bound students from the scientific students except length of study. The twelfth year program and advanced placement is studied almost exclusively by students with a scientific bent.

7. The training of teachers (both inservice and preparatory) has been reformed to include the study of analysis, algebra, and geometry from a modern point of view, as well as more years (semester hours) of such study. Generally, a beginning mathematics teacher now graduates into the profession with an average of 30 semester hours of collegiate mathematics. (A semester hour is one class hour of study per week for 15 to 18 weeks). For permanent certification, the teacher is required to take an average of 15 additional semester hours of study in mathematics including further study in abstract and linear algebra, modern geometries, probability, and analysis (or theory of functions).

Due to the recommendations of the committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America and certain accrediting associations, the mathematical requirements for certification of teachers in Elementary Schools has been increased.

8. All of these movements, begun roughly as early as 1951, have had a profound good effect on the interest, attitude, and achievement in mathematics by the students in our schools. Likewise, teachers have found it necessary to increase their knowledge of mathematics by continuing their study of the subject.

9. Due to the improved education in the elementary schools, and the progress in better mathematical training of the teachers of mathematics, what may be described as the second step in reform is now under way. The impetus for this second step

was the Cambridge Report[1] which projected a program from Kindergarten through senior high school for the next generation. This program presented mathematics as a continuous study, comprehensive (not by branches or subjects) from the contemporary point of view, encompassing most of the study that now occurs in the first three years of University work.

A second force was the recommendation of the OECD meeting at Athens, November, 1963, reported in Mathematics Today.[2] Here there was advocated a comprehensive-global rebuilding of the sequence and scope of the secondary school mathematics, much in accord with the Cambridge Report, but not as extensive. It brings Calculus, Elementary Differential Equations, Probability and Numerical Analysis (with Computer Orientation) into the secondary school program. An attempt to construct and put into operation a curriculum of this nature is explained earlier in this report (address given by H. F. Fehr, Monday, December 5) The School Mathematics Study Group is making an investigation into a similar reconstruction.

As a final observation we note a growing interest in applied mathematics. Most reform movements (through undergraduate study) are now investigating means and ways of showing

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[1]Goals for School Mathematics. Educational Services Incorporated, Houghton Mifflin Company, 1963. $1.00.

[2]Mathematics Today, A Guide for Teachers. Organization for Economic Cooperation and Development, 1964, Paris 16, France.

how mathematics intervenes in the Natural, Physical, Behavioral, and Social Sciences, as well as situations in the sciences which can be used to motivate the study of mathematics.

10. The Undergraduate College Program, which has been and still is under study by the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America, has also become fairly stable. The first year and one half of university study is a fairly rigorous study of Mathematical Analysis which comprises continuity and limits, an advanced treatment of analytic geometry, differentiation, integration, linear algebra, and an introduction to the solution of differential equations. The remaining half year, is addressed to the future interest of students in a variety of extensions - such as finite mathematics, numerical analysis, algebra, probability, differential equations, or advanced calculus.

For students not ready to begin this program on entering the college or university, most institutions offer a year of preparatory study (some with college credit, others with no college credit) in algebra, trigonometry, and elementary functions.

URUGUAY

In this report for the second Inter-American Conference on Mathematics Education (Lima 1966), there is presented: a) a review of the activities from 1961 to 1966, b) verification of whether the recommendations of the Bogota conference have been put into practice, c) an evaluation of the extent to which Uruguay particpates in the general reform movement in mathematics instruction. The report is concerned above all with high school instruction, though it will also briefly treat some questions relative to the national university.

High School Instruction

Teaching personnel:

In contrast to most Latin-American countries, Uruguay suffers little population growth. In the past few decades high school instruction has been considerably extended, not without detriment to its quality. About thirty new mathematics teachers are needed in Montevideo every year and approximately the same number for the rest of the country. For the moment it is impossible to satisfy the demand with appropriately trained persons. The university does not prepare intermediate level teachers. The CNES (National Council for High School Teaching) prepares its teachers in the "Artigas" teacher's institute, where about fifteen students register every year, though only four or five graduate. Some regular teaching positions are filled by individual merit or competitive examination or both. Nonetheless, many vacancies remain, which then must be filled with little regard for qualifications. The "temporary teachers" thus employed may teach for many years and enjoy practically the same benefits as regular faculty. In particular, they are regularly promoted on the teaching roster, since rank there is based not on scientific or pedagogical competence, initiative or other virtues but merely on seniority. Some of the temporary teachers have partially or totally finished their university studies; many others are primary school teachers, many of which do not have enough preparation for their duties. Practically none of them have a proper education for becoming good intermediate level teachers without much more special supplementary study. There are presently about the same number of temporary teachers as regular teachers.

It is understandable then, that high school level instruction in mathematics, which was previously quite satisfactory in Uruguay, is now rapidly deteriorating and that students are not involved in vital, interesting and effective learning along the lines of currently accepted orientations of the advanced countries. It must also be pointed out that the best teachers tend to group themselves in the upper of the two cycles of high school instruction. This has produced a clear imbalance between the levels and a very high percentage of failures in the examinations at the upper level. If this situation is not corrected it will have prejudicial repercussions also in the university.

Advanced professional studies:

Shortly before the Bogota conference, CNES had planned the first advanced professional courses for teachers in service at the intermediate level. The courses were given in Montevideo during July of 1962. They lasted for two weeks and were attended by many teachers in the Capitol as well as some from other parts of the country who were granted scholarships by CNES to cover travel and living costs. The courses were given for intermediate level teachers, some of whom were at the same time university teachers; above all they were teachers from the Artigas Institute.

In some aspects these courses had clearly a positive effect: They were at a relatively high level; stimulated among the participants an interest in studying mathematics topics which were new for many of them; and contributed to their awareness of the need and the urgency to improve their education. Nevertheless, the courses had been prepared, at least in part, before the Bogota conference and most of the organizers and lecturers, while well prepared in substance and pedagogy of a traditional nature, were not familiar with the great reform movements that were going on in more advanced countries. Hence, little advance was made in the most desirable direction; traditional geometry had a role that was probably too important. Algebraic structures were not treated with adequate thoroughness and emphasis; and the samemay be said of probability calculus and statistics.

In the following years, up to the present, CNES organized advanced professional courses during vacations. While most of these were generally acceptable from a technical point of view, only a few of them were related to the reform movement, which continues to be unknown or resisted by many CNES teachers, including some whose positions involve important responsibilities in orienting instruction. The lack of connection between the CNES and the university probably contributes to the persistance of this situation

It has certainly been helpful that in the advanced professional courses for teachers lectures have been given by foreign professors such as Marshall H. Stone, John L. Kelley, Lucienne Felix and Georges Papy. Their participation excited a great deal of interest but was almost always too brief to have a profound after-effect.

Observations:

a) The rapid expansion of intermediate level teaching throughout the country is in itself an admirable phenomenon. But it has created difficult problems for the CNES which this body cannot escape and for which proper solutions require changes in internal organization and the establishment of permanent ties with other bodies, principally the university.

b) Although Uruguay's geographical extension and population are relatively small, instruction in the republic is very decentralized and the different branches are governed by autonomous boards or directors. The actual practice however is for institutions to jealously guard their independence and show little disposition to collaborate in the solving of problems. In the past, for example, there has been a tendency to have syllabi developed exclusively by faculty of the respective branches without inviting teachers from other branches to give courses or participate in discussions or study meetings. Also the transfer of students from one division or school to another or from one branch to another is beset by rigid regulations.

c) The CNES has not succeeded in establishing any significant difference between regular and temporary teachers, even in the matter of salaries and promotion. For that reason, few would-be teachers are inclined to enroll in the Artigas Institute, since it is much easier for them to enter the teaching profession as temporary teachers. One should add, also, that the demands of the Artigas Institute may be somewhat severe, as the small number of graduates suggests.

d) A review of the recommendations of the Bogota conference shows that with few exceptions they have not been taken into account by the CNES. This creates a serious situation.

e) Some teachers have attempted to join the reform movement in mathematics teaching but in general have done so on their own. In this regard it is promising that a group has been recently formed of university and intermediate level teachers on a permanent basis for discussing problems and voluntarily giving courses in modern mathematics to high school teachers. It is very important for the CNES to stimulate, amplify and expand the activities of this group.

f) It is curious to note that no special meetings are held in the Artigas Institute for taking up and discussing modern currents in mathematics instruction. Thus almost all the graduates are unaware of the conference, held at Royaumont, Aarhus, Dubrovnik, Athens, etc. It is to be expected that some of the teachers of the Institute should be missing some things; but not that they should ignore the mass of published literature or that being aware of it should not put it in the hands of future teachers.

g) The advanced professional courses for teachers of the CNES constitute a laudable activity but they are of very short duration, do not correspond to systematic programming, and do not take due account of modern tendencies. They should be incorporated into a permanent activity designed to orient in-service teachers and activate the participation of the best of these teachers in gradual reforms properly organized by the CNES in collaboration with the council of elementary education and with the university. According to the recommendations of the conference of Bogota, the university should also be involved in the preparation of new programs and the training of future professionals. To carry out such a task the CNES does not at present have an adequate mechanism. In theory, the control and the orientation of instruction is in the hands of the inspection department. Inasmuch as there are only three mathematics inspectors, it is clear that they cannot effectively control the entire teaching faculty of the country, particularly the temporary teachers. It would seem that the task of control and orientation should be separated , the former being the responsibility of a body of inspectors larger than the present and the second being in the hands of a permanent body made up of mathematics teachers selected by their outstanding experience, their ability, and their pedagogical and scientific interests. They should be informed about the great reform movements in teaching which are going on in the advanced countries; yet they should be sufficiently realistic not to be drawn into excesses or hastiness. They need to have an open spirit of cooperation with university colleagues, especially with those who are in research as well as in teaching.

h) Bearing in mind that within Latin America, Uruguay is small country without unfavorable geographical accidents in ich almost a quarter of the national budget for salaries and penses is given to the various branches of instruction; that ementary and high school education are relatively widespread; at in both the CNES and the university there are enough capable ichers and that in the latter institution a small but competent hematics research center has been developed, it is reasonable assume - given a concentration of effort - that a plan for ining in-service teachers and modernizing the instruction mathematics and the education of future faculty should be izable at a moderate cost.

VENEZUELA

Introduction

Six years ago the teaching of mathematics at the intermediate level would be characterized as follows:

1. With respect to the process of instruction or learning:
   - 1.1 An excessive number of students failed
   - 1.2 Discrepancy between the basic mathematics education received by the graduating students and the requirements considered indispensible by institutions of higher learning for beginning their programs.
   - 1.3 Inadequate training for carrying on life's daily activities and various kinds of vocational work.
   - 1.4 Lack of real interest in the subject on the part of the students which is in direct contrast to the requirements of life today and for the future.
2. With respect to teaching personnel:
   - 2.1 A high percentage of teachers without degrees (approximately 80%)
   - 2.2 A scarcity of instructional materials and poor quality of that which is available.
   - 2.3 Heterogenity of theories of the teaching-learning process.
3. With respect to the administrative structure:
   - 3.1 Little knowledge of the process of mathematics teaching
   - 3.2 Lack of specialized supervision.
   - 3.3 Lack of technical assistance to mathematics teachers.

This situation hass ben favorably changed to some extent by the serious and responsible attitudes which have prevailed in the study of the traditional situation and a search for practical solutions.

Evaluation of Mathematics Instruction in Public High Schools

In the academic year 1959-60, the Office of Secondary and Special Education of the Ministry of Education put the Department of Mathematics and Physics of the Pedagogical Institute of Caracas in charge of making an evaluation of mathematics instruction in the public high schools of Venezuela. This evaluation was completed in 1960 and long and short term recommendations were made, many of which have been welcomed and implemented by competent authorities.

The importance of the investigation lies in the unobjective picture it furnished of the situation in mathematics instruction at the intermediate level which is serving as a base for the changes currently being made. It is thus possible to pay attention not only to the nature of the changes but also to other aspects of mathematical instruction which are equally important.

Consequences of the First Inter-American Conference on Mathematical Education

Venezuela showed great interest in the first Inter-American Conference on Mathematics Education as is corroborated by its delegation, second in size only to that of Columbia where the conference was held. The largest representation came from the

Center University of Venezuela, but it happens coincidentally that almost all of the delegates of that institution also held chairs in the Pedagogical Institute in Caracas. This explains why the content of the conference was mostly disseminated at the Pedagogical Institute. Series of lectures were organized in Caracas and some of the other cities of the country. They stimulated interest in the teaching of the new mathematics. The first Inter-American Conference on Mathematics Education also helped to speed up reform movements in higher education institutes particularly in the Pedagogical Institute of Caracas of the Central University of Venezuela.

Training of In-Service Teaching Personnel

In 1963, the Ministry of Education organized a First National Workshop on Mathematics Instruction through the overall Educational Planning Office (EDUPLAN), of the Office of Secondary High School and Special Education (DESSE) of the Pedagogical Institute of Caracas. Professor Jose Alejandro Rodriguez Ortega coordinated the workshops and the following subjects were taught:

Algebra, Professor Jesus Salvador Gonzalez, assisted by Julia Ramon Riere

Analysis, Professor Mauricio Orellana Chacin, assisted by Delia Flores

Geometry, Professor Eduardo Lima de Saa

Logic, Professor Cesar Castro Ochoa

Lectures for the Workshop, Dr. Paul Dedecker and Professor Jean Overholser.

28 graduate professors who teach in various parts of the country were participants. The course was so successful and had such an impact on the participants that it gave rise to the formation of a working group which produced the work Mathematics, instruction naterial of great value which is the basis of the instructional plan for modern mathematics which is now being started in Venezuela.

During the academic year 1964, the Second National Workshop n Mathematics Education was held, having characteristics imilar to those of the previous ones. Professor Narciso Rodriguez rtega was the coordinator and courses were given following the ume patterns as in 1963. The course was given on two levels. ie first level included 13 teachers, the second 9 teachers and auditors.

In 1966, as a result of the initiative of the Department Mathematics and Physics of the Pedagogical Institute of racas, workshops on mathematics instruction were set up in racas as well as in Barquisimeto. A private corporation, in s case the Shell Foundation, underwrote the costs of the kshop. The Caracas workshop was coordinated by Professor e Alejandro Ridriguez and followed the same pattern as vious workshops. There were 31 graduate and undergraduate chers participating. The data from the Experimental Pedacal Institute in Barquisimeto are not available.

Curriculum Revision

This work is handled by the Overall Educational Planning Office (EDUPLAN) through the plans and programs commission (CPP).

A study of the constitution of the CPP reveals that two important points should be considered before regular activities are begun: The organization of the commission and the facilities at its disposal for the accomplishment of its tasks. At present, the CPP is not a curriculum planning group. Rather it should be understood as a coordinating body and even then it would have to be reinforced by specialists in every major field of curriculum and be able to count on the assistants of experts and consultants and advisors if it is to accomplish its work responsibly..

To date the commission on Plans and Programs has achieved the following:

A. The formulation of the objectives of general education;

B. Preparation of the basic structure of the educational system;

C. Elaboration of the study plan or plans;

D. Selection and organization of content at the primary level;

E. Selection, distribution, and organization of specific objectives and educational experiences by grades;

F. Formulation of the objectives of subjects by grade levels;

G. Selection of concrete and dynamic basic materials that can be suggested for the program: methods, activities, evaluation techniques and situations in which they can be employed, audio visual materials, bibliography for teachers and students.

H. Recommendations for dealing with individual differences;

I. Preparation of supplementary guides for teachers;

J. Preparation, printing and distribution of programs;

K. Implementation of the curriculum.

Implementation of the curriculum is taking place at both the primary and secondary levels in the following form: elementary education: urban zone, six schools (nursery, first grade and second grade); rural zone, seven schools (first, second and third year grades)

ntermediate education: metropolitan zone, three normal schools, ne high school, one industrial school; in other parts of the ountry, two high schools.

The Department of Mathematics and Physics of the Pedagogical nstitute of Caracas commissioned Professor Narciso Rodriguez rtega to advise on and coordinate the work of the commission iarged with formulating the mathematics programs. The work complished to date is of high quality and promises to set motion favorable changes in mathematics instruction.

e Training of Mathematics and Physics Teachers

Mathematics and Physics teachers are trained in the partment of Mathematics and Physics of the Pedagogical Institutes. Venezuela there are two pedagogical institutes, the Pedagogical titute of Caracas, created thirty years ago, and the erimental Pedagogical Institute of Barquisimeto, established en years ago.

Since 1952 the department of Mathematics and Physics of the Pedagogical Institute of Caracas has taken into account the need for change in the content and orientation of instruction in mathematics and physics. This need was further underscored by the conslusions and recommendations of the First Inter-American Conference on Mathematics Education which contributed to gradual modifications of the content of programs. It can be stated today without fear of contradiction, that mathematics and physics teachers who have graduated from the Pedagogical Institute of Caracas since 1961 are capable of assimilating, adopting and initiating the reforms required by the teaching of mathematics. Nevertheless, there still remains to be set up a functional structure in the Department of Mathematics and Physics to assure complete reform.

The Evaluation Commission of the Pedagogical Institute (CEDIP) has just finished a self-study of the Pedagogical Institute. This study provides a firm basis for undertaking a thorough reform. Through such a reform the Department of Mathematics and Physics will be able to acquire a modern and functional structure which will permit it to educate its students along lines and tendencies of present-day teaching of physics and mathematics.

Coordination of Mathematics Instruction

This service, just recently established, promises to achieve uniform criteria for the different aspects of the teaching-learning process of mathematics. The service is under the Office of Secondary, High, and Special Education and is advised by the Department of Mathematics and Physics of the Pedagogical Institute of Caracas. Professors Narciso Rodriguez Ortega and Jose Alejandra Rodriguez are the joint advisers; Professors Luis Jose Marcano and Narciso Rodriguez Ortega are in charge of implementation and execution; and the supervisor of mathematics and physics handles supervision.

PART IV

A. CONCLUSIONS

FOREWORD

This section presents the findings and conclusions of the conference. No conference, of the sort reported here, can have force in the future activities relating to improvement of instruction unless some sort of plan or scheme is laid out for the action. With this in mind, the closing days of the conference were occupied with framing resolutions, charting action for the years ahead, and setting up a sort of machinery to insure that some progress will take place. These activities and conclusions are reported in this part of the report.

For future meetings, for reference, and for general information - the organization of the conference, the program, the participants and the observers - are listed in the Appendix which concludes this report.

THE IACME COMMITTEE 1967 -

During the years 1961 to 1966, an Inter-American Committee on Mathematical Education, appointed at the Bogota Conference in December 1961 acted in the best capacity it could to insure that most countries would become aware of the conclusions of the Bogota conference, and make some efforts to improve the mathematical education of the country. Hampered by lack of funds to have occasional meetings - and aided occasionally by the National Science Foundation which enabled the members to gather at certain international activities of the Foundation - the Committee did the best it could.

The Lima Conference took some steps toward aiding the new Committee to function in a more efficient manner. A sub-committee of the official participants was charged to draw up a modus operandi for the Committee, nominate a new Inter-American Committee, and give it a charge. At the closing session, the sub-committee presented its findings, which after a short discussion was unanimously accepted. The report follows.

I. Basic Policy of the Inter-American Committee on Mathematical Education

Approved at the closing session of the Lima Conference, the 12th day of December, 1966.

A. The Inter-American Committee on Mathematical Education (IACME) originating in the First Inter-American Conference on Mathematical Education, December 4-9, 1961, is a non-governmental body affiliated with the International Union of Mathematicians through the International Commission on Mathematical Education. The purposes of the Committee are to serve as a technical organ in the sense of, and the scope of, the recommendations made at the Conference cited above and at the Second Inter-American Conference on Mathematical Education held at Lima, December 4-12, 1966.

B. In accordance with the decisions of the Lima Conference, the composition of the Committee, until the next conference is held, shall be:

Marshall H. Stone, U.S.A., President
César Abuauad, Chile
Ricardo Losada, Colombia
Manuel Meda, Mexico
Leopoldo Nachbin, Brazil
Luis A. Santalo, Argentina
Juan Jorge Schaffer, Uruguay
Egardo Sevilla, Honduras
José Tola A., Peru

The Committee will assign to its members the duties of vice president, secretary, and such other offices that are deemed necessary. Likewise it is given authority to designate replacements in cases of retirement.

C. Committee membership will require annual minimum dues of $100 per country, paid by a body or organization which (in the judgment of the Committee) would be representative of the activities which they promote in the respective countries.

D. The Committee will solicit the support of organizations and bodies, which by their character and goals, correspond to the objectives of the Committee and the activities which it advocates.

II. At the Close of the Lima Conference, the following resolutions were adopted unanimously and directed to the Ministries of Education, Universities, International Organizations, OEA, UNESCO, AID, and to Foundations supporting teaching and research at all levels of mathematical education.

Considering

a) That although many of the recommendations of the Bogota conference which this conference has re-stated have already been carried out and in view of what has been accomplished and of the encouraging progress already realized, some of the Bogota recommendations can and should be expanded and amplified in detail;

b) That actually there is abundant information and accomplished experimentation to enable the establishment of an ideal program of modern mathematics for intermediate level education;

c) That in Latin America as a whole and to a greater or lesser extend in each separate country, for various reasons, mathematics teachers continue to have an inadequate preparation; that there is a shortage of mathematics teachers who are in a position to write good textbooks and who are able to actively participate in writing syllabi and that all this makes it appropriate to stress to the universities the need to plan efficient and dynamic programs geared toward the awarding of academic degrees in mathematics which will strengthen the development of this science in each country;

d) That it is necessary to take advantage, to a reasonable degree, of the academic resources of the various Latin American universities, in order that the progress of one university may benefit the others and in this way help avoid the migration of young Latin American scientists who for economic reasons go abroad looking for a better scientific atmosphere;

e) That in order to accelerate the rate and efficiency of reform of mathematical education at the secondary level, it is very important to publish textbooks, guides and other bibliographic materials, as well as their diffusion to all the mathematical faculties;

f) That in order to increase the efficiency of IACME it is convenient to assure its perpetuation through representatives in each country;

g) That it is essential to know with precision the possibilities for advanced studies and research that are offered by the Latin American universities, as well as the current state of mathematical education at the different levels in each country;

h) That it is essential to have periodic meetings among mathematics teachers to discuss problems and to set in motion the renewal and intensification of mathematical studies.

It is hereby recommended to

The Ministries of Education,

The Universities and Educational Institutions of each country

The International Organizations such as the OEA, UNESCO, USAID, which have among their goals the development of science and in general,

The Institutions tied to mathematical teaching and research at all levels,

the following:

I. On the Curricula for Intermediate Education

1. That the mathematical programs for secondary education introduce in a sequence and manner in accord with the possibilities of each country, the following topics of the ideal program:

Age 12-15 years:

1. Notion of set. Operation with sets.
2. Relations (functions, equivalency, order, composition)
3. The ring of Integers; powers; divisibility
4. Binary operation. Illustration of the concept of the group, solving equations of the type $a \cdot x = b$; application to geometry and to number systems
5. Progressive and descriptive introduction of the axioms of the geometry of incidence, parallelism, ordering, parallel projection, and translation.

6. Progressive and descriptive introduction to rational and real numbers. The linear and quadratic equations.
7. Vector space of the plane
8. Coordinates; equation of the line; inequalities; half planes; some applications.
9. Forms of representing a function (graphic tabulation, analytic expressions ...); operations with number functions.
10. Metric geometry of the plane; scalar product, Pythagorean theorem.
11. Analytic geometry of orthogonal bases (line, circumference,...);
12. Solving of linear systems of equations.

<u>Age 15-18 years</u>

1. Study of real numbers.
2. Euclidean space; Orthogonal bases; Inequality of Cauchy-Schwarz.
3. Linear transformation of the plane; (2 x 2 matrices); the group of orthogonal transformations; Similarity.
4. Complex numbers
5. Trigonometry
6. Combinatoric analysis; notions of probability
7. Euclidean algorisms; theorem of unique factorization
8. Polynomials; theorem of remainder.
9. Progressive Introduction and description of some topological concepts; Topological spaces used in elementary analysis
10. Continuous functions; limits; series;
11. Derivative of functions of one variable; applications

12. Integration (preferably as the limit of the sum)
13. Special elementary functions (exponentials, logarithmic, circular)
14. Determinants
15. Geometry of space using Euclidean vector space of 3-dimensions
16. Probability and elementary statistics.

That the following observations should be born in mind:

a) It is convenient to try out the new program in its entirety in pilot courses, analysing carefully the result to decide in their light the vocabulary and sequnce of the overall presentation;

b) The present order of topics is not assumed to be the most convenient. They have been presented in a general sequence so that each school or each teacher may have freedom of choice in the order and manner of presentation of the topics;

c) The program is intended for institutions generally called bachillerator designed to prepare future students of universities. For special secondary schools (commercial, industrial, teacher training, etc), some of the topics corresponding to the age bracket 12-15, are thought of as common and necessary to the education of all secondary students;

d) To develop the proposed program presupposes a primary education that gives the student a solid background in the handling of operations in arithmetic and an intuitive knowledge of geometric figures. The modern concepts do not exclude the use of the calculation techniques learned in primary school, which should be exercised continuously by the student to maintain skill.

2. Statistical data should be gathered from each country on the result of the different programs with the possible variables maneuvered, so that the different programs can be evaluated objectively, as to the advantages and disadvantages of each one of them.

3. That the program of mathematics in universities for engineers and students of other branches of applied science be adapted to the needs of the future user, emphasizing a basic curriculum with variants in accordance with the specialty.

II. Concerning the Teaching of Mathematics to High School Students and the Teaching Methods of Primary Schools

4. That the universities pay special attention to the establishment and maintenance of centers active in the teaching at advanced levels and carrying on mathematical research, along with all the factors necessary to fulfill these functions efficiently.

5. The stimulation of multilateral agreements among the universities of the same country or of various countries through which students and teachers of any one of them can take advantage of the academic facilities of any of the others, establishing a system of equivalencies of study programs to facilitate this exchange.

6. That each country make an increased effort to turn out sufficient teachers of mathematics at the high school level, who have an adequate scientific background and whose training has been guided toward their professional duties. That among teachers and professors of mathematics a core group of experts with a sound scientific training, and having the necessary methodological preparation to work efficiently in drawing up programs, write textbooks for students and carrying on educational research.

III. On the Improvement of Inservice Secondary School Teachers

7. That programs be organized, and already existing courses be intensified, to improve inservice teachers of secondary school mathematics, and to this end permanent centers be established in each country, in connection with the universities. Further, for the improvement of inservice teachers all modern technological means for diffusing knowledge on a grand scale, such as radio and television, should be used.

IV. On the Preparation of Texts and other Bibliographic Materials

8. That an effort be made to publish:

a. brief monographs on specific topics intended for secondary school teachers, taking into account the new curriculum;

b. textbooks for high school students as well as the corresponding guides for the teachers;

c. booklets for high school students destined for the extension of current mathematical topics;

d. bulletins with the purpose of diffusing the results of pedagogic experiences on the reforming of mathematical education; to review publications of interest to teachers; and to give account of other important activities;

e. a Latin American magazine to deal especially with topics related to the teaching of mathematics at the high school level.

V. On Diverse Topics

9. That in order to increase coordination, the Inter-American Commission on Mathematical Education patronize the formation of a committee in each country to foment, on a national or regional scale, activities conducive to the development of mathematics; that these committees should also seek the cooperation mentioned in the recommendations of the "Conference of Ministries of Education and Ministries in charge of the Economic Planning in the Countries of Latin America and the Caribbean," held in Buenos Aires from June 20-30, 1966. These committees should be representative of all mathematical activity including investigators and professors at all educational levels.

10. That a guide be prepared and distributed to be as complete as possible of all the Latin American institutions which offer programs of work and study at a high level in the field of mathematics, indicating with exactness the requisites for admission, awarding of degrees, scholarships, exchange students, etc. This information should be kept up-to-date.

11. That competent offices of each country elaborate a census for information on the condition of the development of mathematical education at the elementary, intermediate, and advanced levels.

12. That national congresses of professors of mathematics and secondary school teachers be organized periodically with the participation of university professors, and secondary level teachers invited from other countries as well as professors of disciplines related to mathematics. It would be convenient that in these meetings the research groups on secondary mathematical education also participate.

13. That national or regional symposia be organized periodically with the participation of the graduates, visiting professors and professors and researchers of sciences in which mathematics is a basic tool. These seminars would consider among other activities:

a. drawing up extensive courses on special topics at the graduate and post-graduate levels;

b. the formation of seminars on specialized mathematics topics and on problems of education at the superior level;

c. the presentation and discussion of brief reports on research work.

14. That national societies of mathematics be organized in those countries where they do not now exist, in which teachers of secondary education and higher education would participate in order to promote the improvement of mathematics within the nation and to seek in very way better conditions for the professional exercise by its members.

III. As a matter of information, especially for literally the thousands of interested persons who have not been able to procure the Report of the First Inter-American Conference, the recommendations of the Bogota Conference are reproduced here. These recommendations in consideration with the reports in Part II of this Report, give some indication of their effect in promoting reform in Latin American Countries.

Resolutions

It was the sentiment of the Organizing Committee of the Conference that, in order to bear fruit, the Conference must reach clear, well-rounded conclusions as a result of its deliberations and must make explicit recommendations for putting them into effect. This was interpreted to imply that the Conference must also propose an organized and coordinated effort for carrying out its recommendations in the various countries represented at the Conference. Accordingly, the participants, in plenary sessions, adopted the resolutions that are presented herewith.

The First Inter-American Conference on Mathematical Education

Considering

(a) That, in our technological society, mathematics is a vital branch of knowledge and an indispensable instrument for economic and social progress, particularly through its applications to biology, economics, statistics, physics, chemistry, engineering, etc.;

(b) That an alarming dearth of teachers of mathematics endangers the development of this science and of its applications;

(c) That, consequently, it is urgent to adopt measures to strengthen the training of a large number of qualified teachers, principally for the secondary school level;

(d) That the teaching of mathematics at that level must be entrusted exclusively to teachers who have received a professional training in mathematics in institutions of university rank;

(e) That, as one of the most important requirements of teaching, the teachers of mathematics must keep up to date in their field,

Recommends to the Governments of the Countries of the Participants and to Authorized Agencies of These Governments

I. In Connection with the Training of Teachers

1. That centers for the training of high school mathematics teachers should offer scholarships and other facilities to those students who choose this career and that high school students should be informed, by means of conferences and publications, of the existence of a career as teachers and researchers in this field, and of the social importance and of the possibilities offere to those who follow it.

2. That the training of teachers of mathematics should be the sole responsibility of the university and under the influence of the most competent mathematicians, to avoid the cleavage between the teaching of mathematics and progress in science and technology. In the meantime, where this training is carried out in special institutions, mathematics courses should be of a university level.

3. That in the training of teachers of mathematics in the secondary schools, the courses should be modernized and those of a pedagogical character should be limited to proper proportions.

II. In connection with Teachers in Active Service

4. That regular contact be maintained between high school teachers and university professors, encouraging the former periodically to attend courses for improvement (regular or special), and that the means to achieve this end, such as scholarships at home or abroad, be increased.

5. That steps be taken to raise the socioeconomic level of the secondary school teacher holding a regular certificate, such as:

a) Guarantee tenure

b) Establish basic salaries equal to those of other professions requiring similar or equivalent academic preparation.

c) Establish a system of promotions with its corresponding implications (increase in salary, reduction of working hours, etc.) automatically based on the number of years of service, considering supplementary advantages and taking into account publications and activities aimed at self-improvement.

d) Establish the sabbatical year.

e) Offer the teacher the possibility of a regimen of complete dedication, as a favorable condition necessary to his progress.

6. That a maximum of incentives be assigned (scholarships, compensation, etc.) so that the teachers of the secondary school who are without certificate but are in active service can

obtain one, and therefore can be covered by the system established in article 5 either by completing their university studies or by taking special courses created for this purpose.

III. In connection with the Improvement of Teaching

7. That the realization of courses and the creation of institutes of an experimental character, for trying out new texts and new methods of teaching mathematics, be encouraged.

8. To suggest to the International Union of Mathematicians, UNESCO, and the Organization of American States, to take under consideration the following steps:

a) The intensification of programs of the training of secondary school teachers of mathematics.

b) The dispersion of activities, projects, and publications which have to do with the improvement and modernization of the teaching of mathematics.

c) The publication and distribution of reports, new texts, and translations written for teachers of the secondary school for their use in teaching and in self-improvement.

d) The encouragement of research as an avenue for scientific and technological progress and as a factor in motivating teaching.

e) The creation of an international center for the purpose of collecting and disseminating information that is relative to new experiments and new ideas in mathematics education.

f) The creation of an Inter-American Commission on Mathematics Education, of a permanent character, for the purpose of providing continuity to the projects and ideas discussed in the Conference and to promote action calculated to raise the level and

and efficiency of secondary school and university teaching of mathematics.

9. To promote a wide exchange of information on new ideas in the teaching of mathematics in all countries through national meetings and other international conferences such as the present one.

10. That delegates and participants establish and maintain contact with the authorities of their respective countries, so that effective measures can be taken to put into practice these recommendations.

11. That the following persons serve as a committee pro tempore, until the establishment of a Commission on Mathematics Education in accordance with recommendation 8(f) above:

Marshall H. Stone (U.S.A.), Chairman
Bernardo Alfaro S. (Costa Rica)
Alberto Gonzáles Dominguez (Argentina)
Alfredo Pereira Gómez (Brazil)
José Tola Pasqual (Peru)

IV. Appreciation

At the closing session of the Conference, the following motion was unanimously approved. That it be

Resolved:

1. To thank the Government of Peru, with special mention of the Minister of Education, Dr. Carlos Cueto Fernandini, for the colaboration in the support of the conference.

2. To congratualte the local Organizing Executive Committee represented by its Executive Board

President	José Réategui C.
Vice President	José Luis Krumdieck L.
Secretary	César Carranza S.
Treasurer	Victor Latorre A.
Member	Jorge Mendoza R.
Pro-Secretary	Jorge Sáenz C.

for its meritorious endeavor toward the successful outcome of the conference and to thank them for the consideration given to the needs of all the particpants.

3. To express thanks to the members of the IACME [illegible] Professors Marshall H. Stone, president, Bernardo [illegible] Alberto Gonzales Dominguez, and his IACME alternate, José Babini, Alfredo Pereira Gomez, Carlos Imaz, and José [illegible] Pasquel for their efforts and work accomplished since the Bogotá Conference of 1961.

4. To thank the personnel of the Secretariat and the translation office for the work they did which made possible the reproduction of all the conference documents within the period of the Conference dates.

5. To single out the valuable contribution made by the following professors to the Conference, and the Governments and Institutions which by their financial assistance, made this possible:

1. Georg Papy and the Government of Belgium
2. André Revuz and the Government of France
3. Hans-Georg Steiner of Germany and UNESCO
4. Erik Kristensen of Denmark and UNESCO
5. Pedro Abellanos of Spain and the Ford Foundation.

6. To thank Howard Fehr, Executive Secretary of the Conference for his brilliant colaboration in organizing the conference.

7. To thank those institutions which made it possible to hold the conference for their valuable financial assistance and cooperation as expressed through their representatives:

The Ford Foundation and its representatives, Dr. Rao and Dr. Fraenkel

The National Science Foundation and its representatives, Dr. Max Hellmann and Dr. Jay Davenport

The School Mathematics Study Group represented by Dr. E.G. Begle

The Institute of Pure and Applied Mathematics of the National University of Engineering of Peru, represented by Rector Arq. Santiago Aguito

The Ministry of Education of the Republic of Peru

The Organization of American States represented by Mr. Heitor G. de Souza

The UNESCO and its representative Madame Ann Hunewald

Lima, Peru, December 12, 1966

PART IV

B. CONFERENCE INFORMATION

A. THE CONFERENCE PROGRAM

Saturday, December 3

12:00 - 6:00PM — Receiving and Registration, Hotel Bolivar

Sunday, December 4

9:00 - 5:30PM — Receiving and Registration, Hotel Bolivar

6:30PM — Reception and Informal Presentation of the Participants -- American Room, Hotel Bolivar

Monday, December 5

10:00AM — Opening Session. American Room, Hotel Bolivar

Presiding: Professor José Tola P.

Reform in Mathematical Pedagogy. His Excellency, Dr. Carlos Cueto Fernandini, Peruvian Minister of Education and Honorary President of the Conference.

Response and Work of the Conference. Dr. Marshall H. Stone, President of IACME.

3:00 - 6:00PM — Reform Movements in Mathematics Education

Presiding: Professor Raefel Laguardia

Activities of the OAS in Mathematics. Andres Valeiras

Mathematical Reform in Spain. Pedro Abellanos

Mathematical Progress in Brazil. Osvaldo Sangiorgi

Mathematical Improvements in Chile. César Abuauad

A New Construction of Secondary School Mathematics. Howard F. Fehr

Discussion

Tuesday, December 6

9:00AM - 12:00 New Curricula

Presiding: Professor Alfredo Pereira Gomez

A Turkish Effort to Improve Secondary School Mathematics and Science. Eugene Northrop

The Danish Mathematics Program. Erik Kristensen

Discussion

3:00PM- 6:00PM Teacher Education

Presiding: Professor César Abuauad

Teacher Training in Brazil. Martha M. de Souza D.

Teacher Re-Education in Puerto Rico. Mariano Garcia

New Curriculum and Training of Teachers in Argentina. Renato Völker

Discussion.

Wednesday, December 7

9:30AM - 12:00 Unsolved Problems in Mathematics Education

Presiding: Bernardo Alfaro Sagot

Problems Faced in the Development of Mathematical Research in Latin America. Professor José Tola P.

Problems Faced in Mathematical Reform with Teachers and Curriculum in Latin America. Luis A. Santaló.

Some Observations on the Development of Mathematics in Latin America. Raefel Laguardia

Discussion

3:00PM-5:00PM Engineering and Science

Presiding: Professor Marshall H. Stone

Mathematics Programs for Engineering Education. Carlos Imaz

The Curriculum in Analysis. André Revuz

Discussion

5:00PM Formation of Working Groups (Commissions) and Committees

Commission on the Curriculum: Chairman Eduardo Suger C.

Commission on the Formation of Teachers and Professors: Chairman César Abuauad

Commission on Re-Education of Teachers in Service: Chairman Osvaldo Sangiorgi

Commission on Preparation of Textbooks, Translations, and Other Materials: Chairman Luis A. Santalo

Committee on Recommendations: Chairman Alfredo Pereira Gomez

Committee on Resolutions: Chairman Ricardo Losado

Committee on Nominations for IACME: Chairman Raefel Laguardia

Committee on Elections: Chairman R. D. James

ursday, December 8

9:00AM - 6:00PM Meeting of the Working Groups Hotel Bolivar

10:00AM The Status of Reform in the Teaching of Mathematics in Belgium, 1966. Georg Papy; Ministry of Education Auditorium.

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[Fr]iday, December 9

9:00 - 11:00AM	Preparation of Reports of Working Groups
11:AM	Visit to the Universidad Nacional de Ingenieria
3:00 - 5:00PM	Education of Teachers and Professors

Presiding: Professor E.G. Begle

A Rigorous Teacher Training Program in Western Germany. Hans-Georg Steiner

Discussion

5:00PM — Plenary Session. Reports of Working Groups on Teacher Training.

[S]aturday, December 10

9:00 - 10:00AM — The Basic Curriculum

Presiding: Professor Burton Jones

The Program in Analysis in the Universities of Central America. Eduardo Suger Cofino.

Discussion

10:15AM — Plenary Session. Reports of the Working Groups on the High School and University Curriculum.

3:00-6:00PM — Reports of Commissions and Committees

Presiding: Professor César Abuauad

Plenary Session. Reports of Working Groups. Recommendations for Future Activities.

Sunday, December 11

Meetings of Resolutions and Recommendations Committees.

Monday, December 12

9:00AM-12:30PM Closing Session

Presiding: Professor Marshall H. Stone

Reading, Discussion, and Adoption of Recommendations

Election of Members of New IACME

Preparation of Proceedings of the Conference

Report of Resolutions Committee

Summary Conclusion. Marshall H. Stone

1:30PM Formal Closing Luncheon of the Conference; Country Club Hotel

Address: Mathematical Knowledge for Today. Professor Francisco Miro Quesada, former Minister of Education of the Republic of Peru.

3:00PM Meeting of the New IACME

B. IACME FOR PERIOD 1961-1966

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Professor Howard F. Fehr

Local Executive Secretary of the Conference:

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Caracas, Venezuela

F. NEWLY ELECTED INTER-AMERICAN COMMITTEE ON MATHEMATICAL EDUCATION

At the final session of the conference, December 12, the following New Inter-American Committee on Mathematical Education was elected to serve until a future conference is held. While the chairman of the committee was named, the committee as a whole was empowered to create its own internal structure and modus operandi.

Chairman

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Members

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G. LOCAL ORGANIZING COMMITTEE

Honorary President

Francisco Miro Quesada, Ex-Minister of Education, Republic of Peru

President

Jose Reategui Canga, Universidad Nacional de Ingenieria

Vice-President

Jose Luis Krumdieck, Universidad Agraria, La Molina

Secretary

César Carranza, Universidad Nacional de Ingenieria, Instituto de Matematicas Puras y Aplicadas

Treasurer

Victor Latorre, Universidad Nacional de Ingenieria, Facultad de Ciencias Fisicas y Matematicas

Pro-Secretary

Jorge Sáenz C., Ministry of Education

Members

Jorge Mendoza, Ministerio de Educacion Publica

Jose Ampuero, Universidad Nacional Mayor de San Marcos, Facultad de Ciencias

Antonio Baxerias, Escuela Naval

Oscar Jahnsen, Escuela Naval

Alfredo Miro Quesada, Centro de Instruccion Militar del Peru

Ruben Munoz, Escuela Normal Superior "Enrique Guzman y Valle"

Gerardo Ramos, Universidad Nacional de Ingenieria

Hugo Sarabia, Universidad Catolica, Facultad de Ingenieria

H. LOCAL SPONSORING COMMITTEE

Arqto. Santiago Agurto,	Rector of the Universidad Nacional de Ingenieria
Dr. Mario Samamé	Director of the Escuela de Graduados de la U.N.I.
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